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THESIS

PROJECTING *COSAGE* OUTPUT IN DISCRETE TIME

by

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December 1999

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The Army's Combat Sample Generator (COSAGE) is a two-sided, symmetrical, high-resolution stochastic simulation model that projects the outcome of ground combat between two forces. Blue force is typically a division; Red force size may be scaled from a fraction of a division to a combined arms army. Because COSAGE is high-resolution (many asset types), it requires extensive data preparation time, and because output is the result of 16-20 replications, substantial simulation run-time.

The analytical model implementation of this thesis is developed to economically project ground combat attrition and munitions expenditures beyond the 48-hour period currently modeled in COSAGE. The implementation evaluates Bayesian estimators of time-period survivorship to estimate expected numbers of kills, both friendly and enemy, during the first 48 hours of combat, then extrapolates those estimates in discrete time steps (here 24 hours) beyond 48 hours. The implementation can be used to project COSAGE output for all combat postures in Northeast and Southwest Asia (NEA and SWA respectively).

An application of the current implementation is to support the warfighting Commanders in Chief (CinC) need to create a Phased Threat Distribution (PTD) in accordance with the Capabilties-Based Munition Requirement Process introduced in June 1997.

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PROJECTING *COSAGE* OUTPUT IN DISCRETE TIME

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LIST OF ABBREVIATIONS AND ACRONYMS

CAA	Center for Army Analysis (formerly U.S. Army Concepts Analysis Agency)
CBMR	Capabilites-Based Munitions Requirement Process
CEM	Concepts Evaluation Model
CINC	Commander-In-Chief
COSAGE	Combat Sample Generator
DIA	Defense Intellignece Agency
DOD	Department of Defense
DODI	Department of Defense Instruction
DPG	Defense Planning Guidance
FEBA	Forward Edge of the Battle Area
IPS	Illustrative Planning Scenario
KV	Killer / Victim Scoreboard
MLRS	Multiple-Launch Rocket System
MOE	Measure of Effectiveness
NEA	Northeast Asia
OPLAN	Operation Plan
OTR	Outyear Threat Report
PGM	Precision-Guided Munition
PTD	Phased Threat Distribution
SWA	Southwest Asia

EXECUTIVE SUMMARY

The Army's Combat Sample Generator (COSAGE) stochastically simulates the joint action of combat arms in division-level combat. For example, it can represent artillery, Army aviation, and certain engineering and U.S. Air Force functions in direct support of armor and infantry.

COSAGE is a high-resolution simulation that requires extensive preparation time for each of the various combat postures of interest; its run times typically involve 16-20 replications for each posture for a 48-hour period. The results reported are estimated means (averages) of force types and sizes surviving after a given combat period.

This thesis implements and illustrates a simple discrete-time deterministic model that projects or extrapolates initial 48-hour (2 day) COSAGE results to greater time, determined by posture duration. The implementation also provides a diagnostic tool for checking the validity of COSAGE inputs. The implementation employs COSAGE output-based estimators to estimate the total expected number of kills, both friendly and enemy, and the munitions expenditures, by platform type, required to sustain combat.

The implementation can be used to project numbers of surviving platforms for all postures in Northeast and Southwest Asia (NEA and SWA respectively) beyond the 48-hour period currently simulated in COSAGE. Both the single- and multiple-period models significantly reduce the time required to obtain results beyond 48 hours of simulated ground combat; they run quickly on a personal computer. Additionally, the implementation of the analytical model enables analysts to explore numerous "what if" questions concerning parameters, including fire allocation, firing rates and initial platform numbers.

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I would also like to thank my wife, Elizabeth, and daughter, Lillian, for inspiring me and helping me through the challenge of creating quality work. Without their love and support, this thesis may never have seen the light of day.

Finally, I would like to thank my parents. Everything that I have accomplished I owe to their love and support. This thesis is dedicated in loving memory to my father, Horst M. Schweighofer, the smartest man I will ever know.

I. INTRODUCTION

A. BACKGROUND

Decreasing defense budget funding over the past several years has forced the Department of Defense (DoD) to re-evaluate its spending policies in numerous areas, including that of conventional munitions. As an attempt to curb excessive munitions acquisitions caused by an isolationist "service-centric" approach to munitions requirements generation by the services, the Under Secretary of Defense for Acquisition and Technology introduced Department of Defense Instruction (DODI) 3000.4, Capabilities-Based Munition Requirement (CBMR) Process, in June 1997.

In order to generate battle scenarios for each potential conflict, each service utilizes the Outyear Threat Report (OTR) and Defense Planning Guidance (DPG), two documents dictated by the CBMR Process (Widdowson, 1998). The OTR is produced every odd-numbered year by the Defense Intelligence Agency (DIA), and outlines the DIA's estimate of potential threats for a specified planning horizon (the "outyears") (DODI 3000.4). The DPG, which is published by the Secretary of Defense, outlines national security objectives and policies, military priorities, and projected resource levels available to meet those objectives for the effective DPG period. Included in the DPG are several Illustrative Planning Scenarios (IPSs), which outline hypothetical conflicts, providing a baseline for potential military challenges within the effective years of the DPG. The CBMR Process requires each warfighting CinC, for each given scenario, to produce a Phased Threat Distribution (PTD), identifying friendly forces to which enemy forces (platforms, installations, organizations) are to be assigned for attrition. DODI 3000.4 defines the PTD as

> the CINC's phased assignment of a portion of the enemy's total combat capability (i.e., forces, installations, and organizations) to DOD Component commands. The distribution is a percentage by type of target (e.g., tanks and fighters) by operation plan phases.

The resulting PTD allows each service to estimate the threat it must be prepared to overcome. However, if each service attempts to cover all threats presented for each scenario, munitions over-acquisition is inevitable. It is therefore imperative that targets be equitably apportioned between the available services (Widdowson, 1998).

Once an equitable distribution of enemy targets with minimal overlap of target assignments among the services is created, it is then up to each service to model the ensuing battle. In order to ensure that the applicable Operation Plan (OPLAN) can be successfully completed, each service must generate models to estimate the expected attrition of both friendly and enemy forces, as well as munitions required to meet the assigned goal.

B. THE PROBLEM AND A MODELING APPROACH

Each service historically relies on its own warfighting simulations to determine munitions requirements for each scenario. One of the Army's models, COSAGE (which stands for Combat Sample Generator), is a two-sided, symmetrical, high-resolution stochastic simulation model of combat between two forces, Blue (friendly) and Red (enemy) (U.S. Army Concepts Analysis Agency [CAA], 1993). Running COSAGE requires months of data collection, which is extremely manpower-intensive and expensive, and requires days to run. The purpose of this thesis is to provide Army and joint service analysts with a spreadsheet-based implementation, called DTAM (which stands for Discrete Time Analytical Model), of an analytical model (see Gaver and

Jacobs, 1999) using COSAGE summary output that will quickly and easily estimate expected ground kills and the associated expected Army munitions expenditures for an extended scenario. The spreadsheet is an implementation of an analytical model developed by D.P. Gaver and P.A. Jacobs at the Naval Postgraduate School (Gaver and Jacobs, 1999). Using COSAGE summary data from a 48-hour run as input, the DTAM tool projects future kills for discrete-time (1, 2, or 3-day) time steps, referring to user-defined stopping rules, eliminating the need for additional long, costly COSAGE runs.

C. PROPOSED PROCEDURE

This thesis develops an implementation called DTAM of a deterministic model to predict the munitions required to sustain combat using output from the Army's *Combat Sample Generator* (COSAGE) model; the latter produces results in discrete time (48 hours), so the present model is matched to this data, then extrapolated beyond 48 hours, here in 24-hour (1 day) steps. Parameters of the deterministic model are estimated from summary COSAGE 48-hour output presented in the Killer-Victim (KV) Scoreboards.

The COSAGE summary output KV Scoreboard provides the following information:

1. Combat Posture
2. Shooter
3. Weapon System
4. Munition Fired
5. Target
6. Shots (direct and indirect averaged over 16 or 20 replications)
7. Kills (averaged over 16 or 20 replications)
8. Kills on Dead Targets (averaged over 16 or 20 replications)

The software implements a series of Bayesian estimators to estimate the expected number of kills, both friendly and enemy, and the expected munitions expenditures, by platform type, required to produce these kills from the COSAGE summary output.

In addition to providing a tool for quickly projecting casualties and munitions expenditures, the analytical model allows "what-if" analysis to be conducted on various platform and posture combinations. The proposed process is depicted in Figure 1.1.

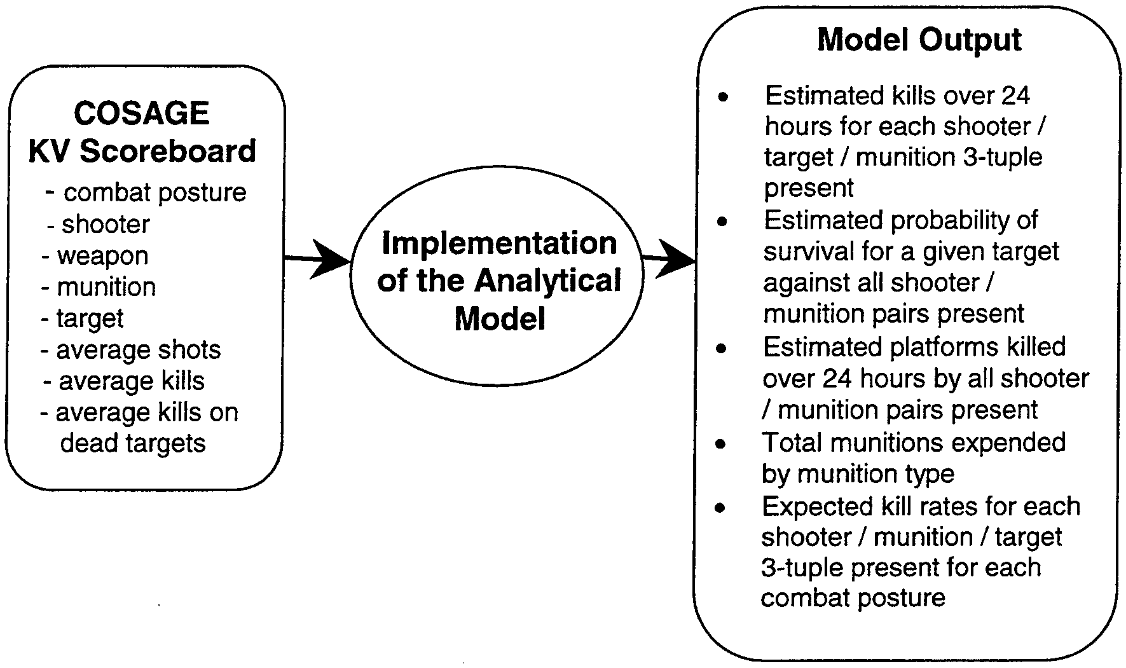

Figure 1.1 The software implements a series of Bayesian estimators to estimate the expected number of kills, and the associated expected munitions expenditures required to produce these kills from COSAGE summary output. The analytical model enables analysts to quickly generate reasonable munitions requirements for each scenario presented.

The analytical model for both single and multiple periods extends the application of COSAGE for all postures in Northeast and Southwest Asia (NEA and SWA respectively) beyond the 48-hour period currently simulated. The analytical model enables analysts to quickly generate reasonable expected munitions requirements for each of the scenarios presented, on a mean basis. *No account is given here of the likely random variability of actual combat, or risk of ammunition shortages.* These issues are saved for future work.

D. MEASURES OF EFFECTIVENESS

Once the analytical model is implemented for the initial 48-hour period using summary COSAGE output, the spreadsheet tool's estimated mean numbers of platforms killed is compared to COSAGE average numbers of kills observed by target type as listed above. A measure of effectiveness used to evaluate the software's plausibility is *the percent error in attrition calculations produced by using the Bayesian estimates.* Once percent error is calculated, an additional, graphical measure of effectiveness is produced by fitting a linear regression of estimated kills produced by the analytical model on actual kills observed in COSAGE summary output, as depicted for the posture NEA "D" (Red Attack vs. Blue Delay) in Figure 1.2. Linear regressions for all combat postures modeled by COSAGE are included as Appendix A. The model fit appears satisfactory for the first 48 hours of combat.

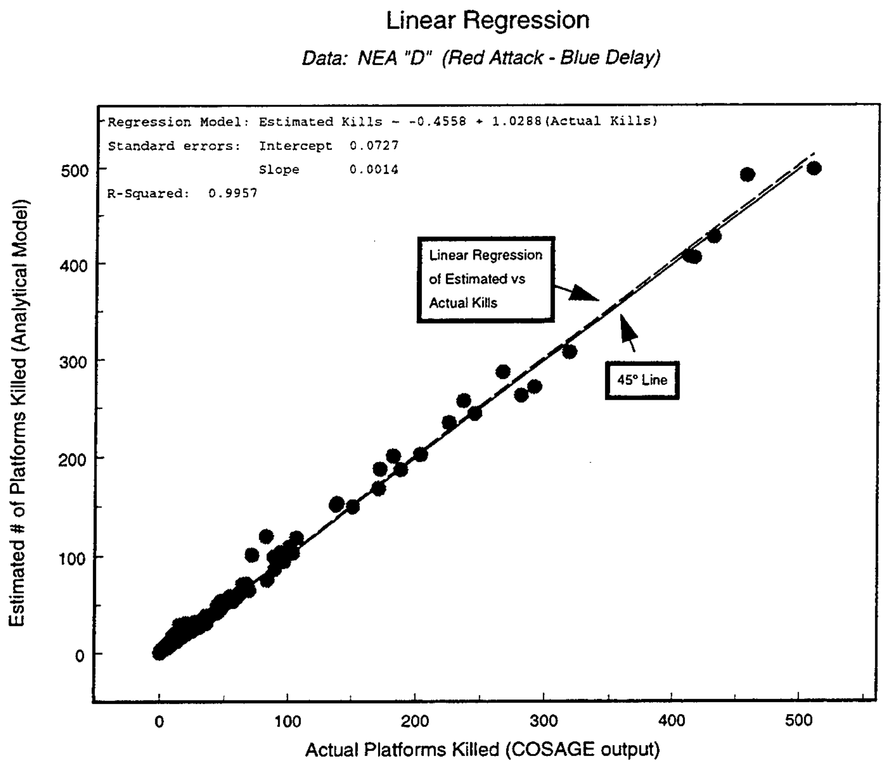

Figure 1.2 A graphical measure of effectiveness is produced by fitting a linear regression of estimated kills produced by the analytical model for 48 hours on actual kills observed in the same 48 hours of COSAGE summary output. Linear regressions for all other combat postures can be found in Appendix B.

E. CONSIDERATIONS

It is important to note that the analytical model is limited in scope. The estimates of expected kills and munitions expenditures presented are based on COSAGE summary output, and therefore do not represent platform interactions for shooter / munition / target 3-tuples not modeled in COSAGE, nor can the model correct any potential simulation deficiencies produced within COSAGE. Additionally, the analytical model is designed to model only Army ground combat of the type presented in COSAGE, and is therefore restricted to the following combat postures presented in COSAGE for either Northeast Asia (NEA) or Southwest Asia (SWA):

1. I - Red Attack / Blue Prepared Defense
2. H - Red Attack / Blue Hasty Defense
3. F - Blue Attack / Red Prepared Defense
4. N - Blue Attack / Red Hasty Defense
5. D - Red Attack / Blue Delay
6. L - Defense Light (Less Intense Static)
7. P - Prep for Attack (Heavy Static)

It is also very important to remember that, while the analytical model produces expected munition expenditures, it does so for (simulated) actual combat only. As with the PTD model, the analytical model does not address "training ammunition, strategic stockpiles, [or] those munitions required to ensure that forward-deployed forces are adequately armed" (Widdowson, 1998).

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II. MODELING APPROACH

A. PROBLEM STATEMENT

Once a PTD is developed and each service is presented its target set in accordance with its role in the applicable OPLAN, the service must predict its munitions requirements based on expected attrition. A tool to approach this problem for the Army is as follows. A Bayesian estimate for the probability with which each target survives a particular munition fired by a particular shooter is obtained from the summary COSAGE output. These estimates are then used to obtain estimates of the expected numbers of targets surviving, expected numbers of targets killed, and expected munitions expended during a 24-hour period. The estimated survival probabilities for each shooter / munition / target 3-tuple for the 24-hour period are then used together with a user-input combat posture to project the numbers of targets surviving for each side through time (24-hour intervals) for a particular scenario.

B. MODELING ASSUMPTIONS

Several assumptions are made in the formulation of the analytical model which is implemented in the software. While the ultimate goal is to capture as much reality in battlefield interactions as possible, some assumptions are required to make the model complete. Additionally, simplifying assumptions provide a manageable model with usable results. Users who rely on the analytical model must be comfortable that the model usefully approximates reality.

The following assumptions are made in the formulation of the analytical model:

1. COSAGE Usefully Models Combat for Particular Postures

Since the deterministic model uses COSAGE summary output, it is assumed that the COSAGE model usefully approximates ground combat. Hence, the deterministic model is subject to all of the assumptions of COSAGE (e.g., ammunition supply is unconstrained).

2. All Forces Operate Along a Single, Continuous Forward Edge of the Battle Area (FEBA)

This simplifying assumption is made to prevent the need to model separate combat postures for different battlefield areas. If combat posture in the multiple-period model is treated as a piston model, similar to that in CEM VI, the entire battlefield is governed by only one combat posture (CAA, 1985). As a result, shooters are provided a uniformly-distributed target set that, although possibly larger than that in discontinuous FEBA models, eludes the difficulties associated with *explicitly* modeling flanks exposed to enemy fire. The current approach *implicitly* models more complicated situations as a simplified force-on-force combat.

3. All Combat Postures Extend No Less Than 24 Hours

Because COSAGE data output is based on a single 48-hour period, and the analytical model on a 24-hour period, it is necessary to assume that the user-prescribed combat posture extends throughout the prescribed modeling period, and, as stated above, across the entire FEBA.

4. All Forces Have COSAGE Battlefield Information

Because the analytical model uses the COSAGE Killer-Victim Scoreboard as its input, the model inherits the battlefield information characteristics generated by COSAGE. Target selection criterion, including target priorities represented by

COSAGE, dictate which shooter / target pairings exist as input to the analytical model (CAA, 1993).

5. Combat for a Blue Division-Sized Force Versus a Red Force Can be Modeled and Used as the Basis for Theater Force Combat

This assumption is made by COSAGE. It is assumed that, by modeling battles at the Blue division level, the model will represent the essence of combat between Blue and Red forces, including key interactions between forces in various combat postures (CAA 1993). Once this assumption is made, it is extended to include the assumption that similar interactions witnessed during combat are scaleable according to force sizes (Blue and Red).

6. Platform Regeneration and Reinforcement Are Not Modeled

In order to simplify the idea of available combat forces, the analytical model does not currently model platform regeneration or reinforcement, although this is feasible (COSAGE models reinforcement, but not regeneration). Thus, the model does not include the logistics pipeline associated with platform repair. Additionally, the model does not explicitly include the effects of such intangibles as crew fatigue and morale (few models do).

7. Individual Shooter Shooting Rate Does Not Change as Combat Progresses

The effective rate at which individual shooters fire their weapons at specific target types is derived from the initial 48 hours of combat generated by COSAGE. This rate is assumed to prevail throughout the remainder of combat. There is no present basis for making any other assumption, but closer examination of combat dynamics could well suggest an improvement.

8. Once a Shooter Designates a Target Set, No Fire Re-Allocation Occurs Based on Target Numbers

Once the estimated individual shooter rate of fire is calculated for each shooter / munition / target 3-tuple in each combat posture, it remains constant. Because estimates of expected kill rate are relatively small for any 24-hour period, the model provides that no target type will be completely attrited to zero, though some approach zero (to a very small order or magnitude); the practical implication is that these *are* zero. In the multiple-period model, although the rate of fire for each individual shooter / munition pair remains constant, the *instantaneous* (actually daily) rate of fire against a given target type is diminished by reductions in the shooter numbers. If the number of a particular target surviving is reduced to zero, the weapons that would fire at it are not reallocated to other targets. Hence, since fire is not re-allocated to different target types; the analytical model does not model fire re-allocation.

9. Each Shooter Has a Non-Increasing Target Supply

The assumption of a diminishing target base allows for the notion of an expected kill rate for each shooter / target pair based on current availability. While not all shooter / target pairs may be available at all times, it is assumed that each shooter will attack those targets present, and that COSAGE usefully models the target acquisition process.

10. Ammunition Supply is Unlimited

As in COSAGE, ammunition supplies are assumed to be unlimited, and battlefield equipment such as tanks begin each battle with a full, inexhaustible onboard munitions load. By making this assumption, the model avoids the possibility of posture transitions based on factors (possibly realistic) other than available combat forces. Additionally, this

assumption retains the concept of an effective kill rate for each target / munition / target 3-tuple present.

11. Some Indirect Fire Weapons are Treated as Direct Fire Weapons

Based on an interview with Professor Sam Parry at the Naval Postgraduate School (Parry, 1999), various indirect fire weapons and their munitions may be treated as direct fire systems. Based on this information, the analytical model implementation treats Multiple-Launch Rocket Systems (MLRS), listed as platform "UMLRS," and Precision-Guided Munitions (PGM), listed as munition "PGM," as direct fire weapons. It is possible that, when summed, average shots attributed to a particular target may exceed the average total shots recorded for the given munition. As a result of this assumption, in the event that COSAGE reported kills exceed shots fired, the implementation sets kills equal to the number of shots fired. This assumption is temporary and convenient, though not easily defensible, and should be resolved in future work.

12. Mines are Treated as One Platform Firing Indirect Fire Munitions

COSAGE reports minefields and fired mines as kill-producing platforms with an initial number set to zero. Since minefields and fired mines produce positive kills, the analytical model treats them as one platform firing indirect fire munitions in order to calculate plausible kills attributable to mines. The analytical model does not currently model mine-clearing operations, except as reflected in the COSAGE output.

C. DATA SOURCES

COSAGE summary reports, provided by the Center for Army Analysis (CAA), provide all the data used in the analytical model. No data require dedicated collection

efforts; the data are also used by various existing higher-resolution theater level simulation models, including CEM VI (Appleget, 1995).

COSAGE is a division level stochastic simulation designed to provide planners data to calculate ammunition expenditures and equipment losses, and to provide equipment losses and munitions expenditures for use in simulation postprocessors (Appleget, 704). COSAGE is used primarily as a method to calibrate data for theater level models, such as CEM VI. COSAGE input includes organizational structures, personnel and equipment strengths, weapon types, numbers of forces and their characteristics, and types, quantities and technical characteristics of munitions for all forces (CAA, 1993). As listed in Chapter I, COSAGE output utilized in the analytical model consists of the killer / victim scoreboards, which provide data on posture, shooter / munition / target 3-tuples present, average shots, average kills and average kills on dead targets.

III. THE SINGLE-PERIOD MODEL

A. DISCUSSION

The single-period model, created by Gaver and Jacobs (1999), is a discrete-time formulation based on a series of Bayesian survival probability estimates for both direct and indirect fire cases (see Section 3). As stated earlier, Bayesian estimates for the probability a target survives munitions fired by a particular shooter are obtained from the summary COSAGE output. These estimates are then used to obtain the following:

1. Estimated kills over 24 hours for each shooter / munition / target 3-tuple present in the given scenario.
2. Estimated probability of survival for a given target against all shooter / munition pairs present.
3. Estimated platforms killed over 24 hours by all shooter / munition pairs present.
4. Estimated total munitions expended by munition type.
5. Expected kill rates for each shooter / munition / target 3-tuple present for each combat posture.

The single-period model is implemented in Visual Basic, and is executed via macros from within the associated Microsoft Excel workbook. A separate Excel workbook, containing all computations for either NEA or SWA, is created for each prescribed combat posture. Because each combat posture formulation relates so closely to its COSAGE summary data source, typical scenario runs generate an Excel workbook containing approximately 170,000 entries. A Pentium II running at 400 megahertz with 96 megabytes of RAM requires approximately 20 minutes to perform all calculations required to produce the estimates listed above for each combat posture scenario.

B. VARIABLE CONVENTIONS

In order to aid understanding of the single-period model outlined below, as well as the full single-period model formulation (Gaver and Jacobs, 1999), all state variables will be referenced in accordance with the following conventions:

1. Although each COSAGE data set is the result of either 16 or 20 replications, all references to COSAGE output assume an average value over all replications.

2. COSAGE summary output is produced for a 48-hour period. Unless otherwise indicated, all values and estimates generated by the single-period analytical model represent a 48-hour period.

3. The time index listed in single-period state variables as 0 indicates the 48-hour period modeled by the single-period implementation. In the multiple-period implementation, the time index 0 will be replaced by *t*, which indicates the various force element counts at the number of days predicted (e.g., 1, 2, etc.).

4. All indices indicating actual forces consist of a single uppercase character (e.g., B indicates Blue forces, R indicates Red forces).

5. All indices with compound subscripts will list subscripts according to the following convention:
   a The first subscript indicates the *firing* force.
   b The second subscript indicates the *receiving* force.
   (e.g., N_{BR} indicates shots by Blue forces against Red forces)

C. STATE VARIABLES

$B(j_B,t)$	Number of Blue platforms of type j_B at time t
$R(j_R,t)$	Number of Red platforms of type j_R at time t

D. COSAGE DATA FOR THE 48-HOUR CAMPAIGN

Reps	Number of Replications indicates the number of COSAGE replications run for the current posture

$\overline{n}_{BR}\left(j_B, w_B, j_R, 0\right)$

Shots (Blue on Red – direct fire)

indicates average number of shots taken over all replications by one Blue shooter of type j_B firing weapon w_B against all Red targets of type j_R as indicated by COSAGE for the initial 48-hour period

$\overline{n}_{RB}\left(j_R, w_R, j_B, 0\right)$

Shots (Red on Blue – direct fire)

indicates average number of shots taken over all replications by one Red shooter of type j_R firing weapon w_R against all Blue targets of type j_B as indicated by COSAGE for the initial 48-hour period

$\overline{N}_{BR}\left(j_B, w_B, j_R, 0\right)$

Shots (Blue on Red – direct fire)

indicates average number of shots taken over all replications by all Blue shooters of type j_B firing weapon w_B against all Red targets of type j_R as indicated by COSAGE for the initial 48-hour period

$\left\{\overline{N}_{BR}\left(j_B, w_B, j_R, 0\right) = B(j_B, 0)\cdot \overline{n}_{BR}\left(j_B, w_B, j_R, 0\right)\right\}$

$\overline{N}_{RB}\left(j_R, w_R, j_B, 0\right)$

Shots (Red on Blue – direct fire)

indicates average number of shots taken over all replications by all Red shooters of type j_R firing weapon w_R against all Blue targets of type j_B as indicated by COSAGE for the initial 48-hour period

$\left\{\overline{N}_{RB}\left(j_R, w_R, j_B, 0\right) = R(j_R, 0)\cdot \overline{n}_{RB}\left(j_R, w_R, j_B, 0\right)\right\}$

$N_{BR}\left(j_B, w_B, j_R, 0\right)$

Shots (Blue on Red – direct fire)

indicates total number of shots taken over all replications by all Blue shooters of type j_B firing weapon w_B against all Red targets of type j_R for the initial 48-hour COSAGE period

$\left\{N_{BR}\left(j_B, w_B, j_R, 0\right) = \overline{N}_{BR}\left(j_B, w_B, j_R, 0\right)\cdot Reps\right\}$

$N_{RB}\left(j_R, w_R, j_B, 0\right)$

Shots (Red on Blue – direct fire)

indicates total number of shots taken over all replications by all Red shooters of type j_R firing weapon w_R against all Red targets of type j_B for the initial 48-hour COSAGE period

$\left\{N_{RB}\left(j_R, w_R, j_B, 0\right) = \overline{N}_{RB}\left(j_R, w_R, j_B, 0\right)\cdot Reps\right\}$

$\overline{S}_{BR}(j_B, w_B, 0)$ Shots (Blue on Red – indirect fire)

indicates average number of indirect shots taken over all replications by all Blue shooters of type j_B firing weapon w_B against all Red targets as indicated by COSAGE for the initial 48-hour period

$\overline{S}_{RB}(j_R, w_R, 0)$ Shots (Red on Blue – indirect fire)

indicates average number of indirect shots taken over all replications by all Red shooters of type j_R firing weapon w_R against all Blue targets as indicated by COSAGE for the initial 48-hour period

$S_{BR}(j_B, w_B, 0)$ Shots (Blue on Red – indirect fire)

indicates total number of indirect shots taken over all replications by all Red shooters of type j_R firing weapon w_R against all Blue targets for the initial 48-hour COSAGE period

$\left\{S_{BR}(j_B, w_B, 0) = \overline{S}_{BR}(j_B, w_B, 0) \cdot Reps\right\}$

$S_{RB}(j_R, w_R, 0)$ Shots (Red on Blue – indirect fire)

indicates total number of indirect shots taken over all replications by all Red shooters of type j_R firing weapon w_R against all Blue targets for the initial 48-hour COSAGE period

$\left\{S_{RB}(j_R, w_R, 0) = \overline{S}_{RB}(j_R, w_R, 0) \cdot Reps\right\}$

$\overline{K}_{BR}(j_B, w_B, j_R, 0)$ Kills (Blue on Red)

indicates average number of Red targets of type j_R killed over all replications by Blue shooters of type j_B firing weapon w_B for the initial 48-hour COSAGE period

$\overline{K}_{RB}(j_R, w_R, j_B, 0)$ Kills (Red on Blue)

indicates average number of Blue targets of type j_B killed over all replications by Red shooters of type j_R firing weapon w_R for the initial 48-hour COSAGE period

$K_{BR}(j_B, w_B, j_R, 0)$ Kills (Blue on Red)

indicates total number of Red targets of type j_R killed over all replications by Blue shooters of type j_B firing weapon w_B for the initial 48-hour COSAGE period

$\left\{K_{BR}(j_B, w_B, j_R, 0) = \overline{K}_{BR}(j_B, w_B, j_R, 0) \cdot Reps\right\}$

$K_{RB}\left(j_R, w_R, j_B, 0\right)$ Kills (Red on Blue)

indicates total number of Blue targets of type j_B killed over all replications by Red shooters of type j_R firing weapon w_R for the initial 48-hour COSAGE period $\left\{K_{RB}\left(j_R, w_R, j_B, 0\right) = \bar{K}_{RB}\left(j_R, w_R, j_B, 0\right) \cdot Reps\right\}$

E. SINGLE-PERIOD MODEL EXPLANATION

The following introduces the single-period model user to the format of the model as it is implemented in Microsoft Excel™ 97. The complete Visual Basic code for the single-period model implementation is included as Appendix B.

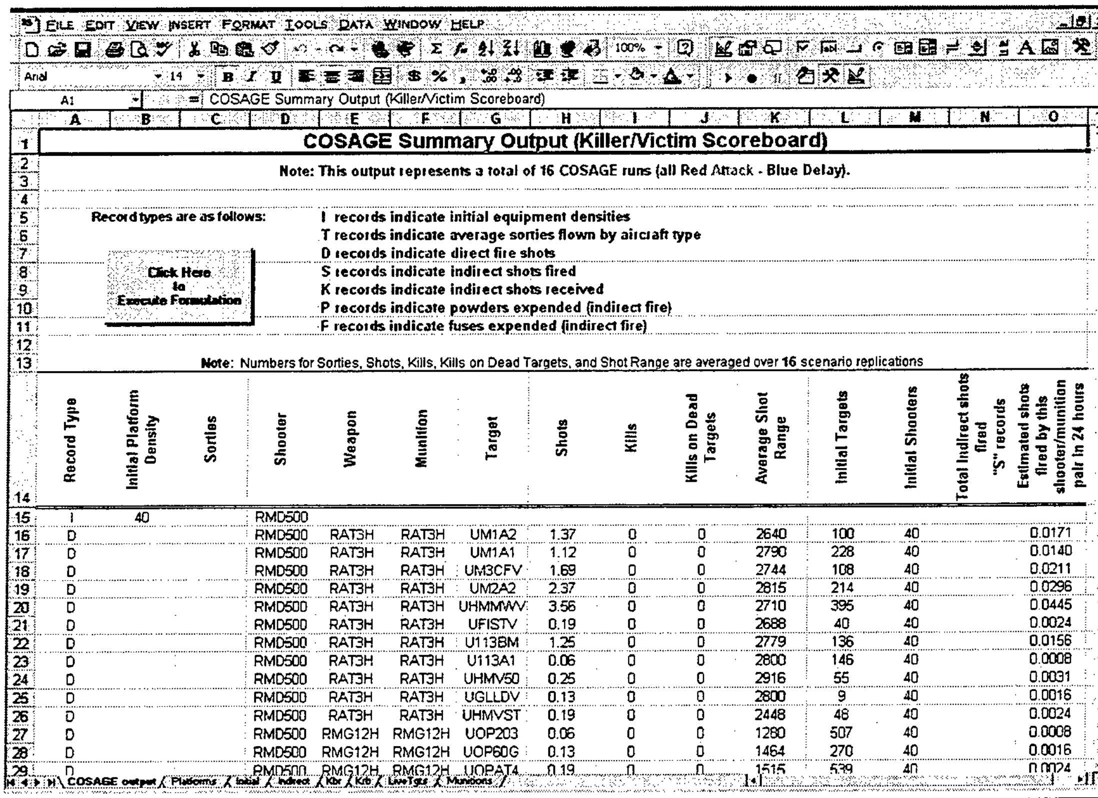
1. Excel Workbook Format

The single-period model for each of the seven combat postures, for both NEA and SWA, is implemented in a separate Excel workbook. For illustrative purposes, all Excel worksheets included in this section represent the NEA "D" (Red Attack vs. Blue Delay) combat posture. Each workbook consists of the following eight worksheets, which are explained in the next section:

a. COSAGE output
b. Platforms
c. Initial
d. Indirect
e. Kbr
f. Krb
g. LiveTgts
h. Munitions

In the first worksheet of the Excel workbook, "COSAGE output," the user will find information concerning data from the COSAGE summary output, including the combat posture modeled by the active workbook (see Figure 3.1). As depicted in Figure 3.1, the second line of the worksheet COSAGE output indicates the combat posture modeled, as well as information concerning the number of COSAGE replications run to

produce the output shown. Additionally, included on the first worksheet of each workbook, COSAGE output, is a command button labeled "Click Here to Execute Formulation" which, when clicked, will execute the Visual Basic code required to carry out all calculations for estimated survival and kill probabilities and munitions expenditures outlined in Section 3, and to populate the entire workbook. Since all calculations are executed when the "Execute" button is clicked, the user needs only to click it once for each new combat posture. By using the command button, the user is freed from the task of running each Visual Basic macro individually, and the potential error of executing a macro before prior worksheets have been populated with data required therein.



COSAGE Summary Output (Killer/Victim Scoreboard)

Note: This output represents a total of 16 COSAGE runs (all Red Attack - Blue Delay).

Record types are as follows:
I records indicate initial equipment densities
T records indicate average sorties flown by aircraft type
D records indicate direct fire shots
S records indicate indirect shots fired
K records indicate indirect shots received
P records indicate powders expended (indirect fire)
F records indicate fuses expended (indirect fire)

Click Here to Execute Formulation

Note: Numbers for Sorties, Shots, Kills, Kills on Dead Targets, and Shot Range are averaged over 16 scenario replications

Record Type	Initial Platform Density	Sorties	Shooter	Weapon	Munition	Target	Shots	Kills	Kills on Dead Targets	Average Shot Range	Initial Targets	Initial Shooters	Total Indirect shots fired "S" records	Estimated shots fired by this shooter/munition pair in 24 hours
I	40		RMD500											
D			RMD500	RAT3H	RAT3H	UM1A2	1.37	0	0	2640	100	40		0.0171
D			RMD500	RAT3H	RAT3H	UM1A1	1.12	0	0	2790	228	40		0.0140
D			RMD500	RAT3H	RAT3H	UM3CFV	1.69	0	0	2744	108	40		0.0211
D			RMD500	RAT3H	RAT3H	UM2A2	2.37	0	0	2815	214	40		0.0296
D			RMD500	RAT3H	RAT3H	UHMMWV	3.56	0	0	2710	395	40		0.0445
D			RMD500	RAT3H	RAT3H	UFISTV	0.19	0	0	2688	40	40		0.0024
D			RMD500	RAT3H	RAT3H	U113BM	1.25	0	0	2779	136	40		0.0156
D			RMD500	RAT3H	RAT3H	U113A1	0.06	0	0	2800	146	40		0.0008
D			RMD500	RAT3H	RAT3H	UHMV50	0.25	0	0	2916	55	40		0.0031
D			RMD500	RAT3H	RAT3H	UGLLDV	0.13	0	0	2800	9	40		0.0016
D			RMD500	RAT3H	RAT3H	UHMVST	0.19	0	0	2448	48	40		0.0024
D			RMD500	RMG12H	RMG12H	UOP203	0.06	0	0	1280	507	40		0.0008
D			RMD500	RMG12H	RMG12H	UOP60G	0.13	0	0	1464	270	40		0.0016
D			RMD500	RMG12H	RMG12H	UOPAT4	0.19	0	0	1515	539	40		0.0024

COSAGE output / Platforms / Initial / Indirect / Kbr / Krb / LiveTgts / Munitions

Figure 3.1 Each Excel workbook consists of eight worksheets, which are accessed by clicking on the tabs labeled "COSAGE etc," etc. along the bottom of the Excel screen.

2. Excel Worksheets

This section is designed to familiarize the user with each of the eight worksheets included in the Excel workbook, including a brief explanation of the key fields used in the analytical model.

a. COSAGE Output

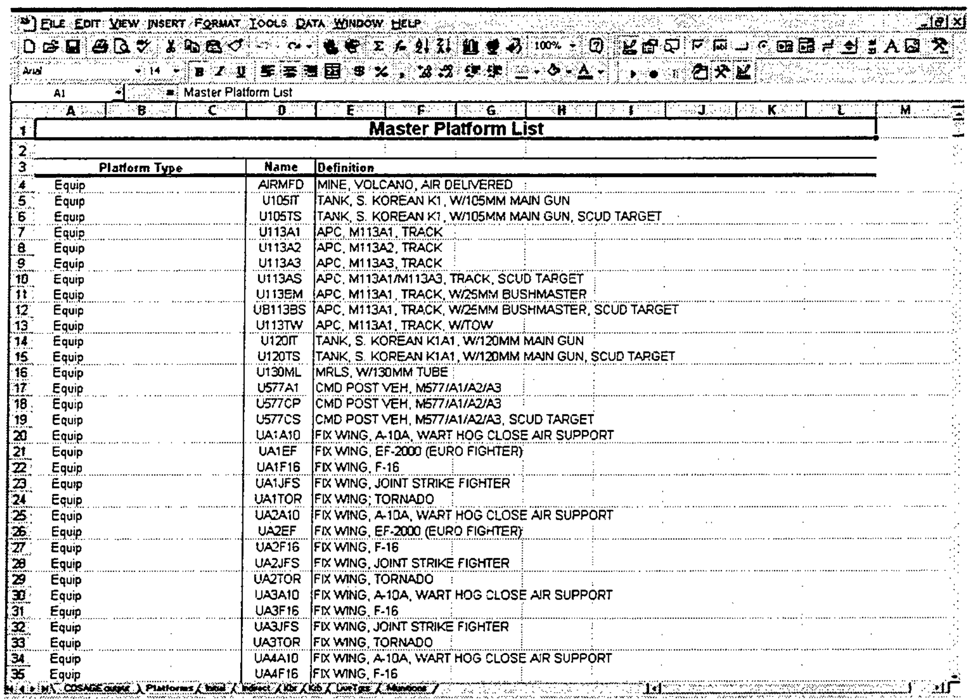
This worksheet displays the COSAGE summary data used in the analytical model, and is the Excel equivalent to the Unix-based summary output produced by CAA for use in theater models, with the following additions (see Figure 3.1). Columns L through N, which include data concerning initial platform numbers and totals for indirect shots fired, as well as column O, which estimates total shots fired over 24 hours by each shooter / munition pair present, are populated once the formulation is executed. The estimate of total shots fired over 24 hours by each shooter / munition pair (column O) is then used by the analytical model in successive time periods.

b. Platforms

The "Platforms" worksheet provides the user with a list of all platforms, weapon systems and munition types used in COSAGE (see Figure 3.2). The section "Name" lists abbreviations used by COSAGE to describe each platform type outlined above, and is followed by the platform description in "Definition." Of special note is the convention used by COSAGE to delineate between Blue and Red platform types. All force-specific platform names, following COSAGE naming standards, begin with either an "R," indicating Red forces, or "U," indicating Blue (or allied) forces.

c. Initial

The worksheet labeled "Initial" provides the analytical model a list of all platforms modeled in the prescribed COSAGE combat posture, as well as the initial number of each platform present in the "Initial Platform Density" section (see Figure 3.3).



Master Platform List

	Platform Type	Name	Definition
4	Equip	AIRMFD	MINE, VOLCANO, AIR DELIVERED
5	Equip	U105IT	TANK, S. KOREAN K1, W/105MM MAIN GUN
6	Equip	U105TS	TANK, S. KOREAN K1, W/105MM MAIN GUN, SCUD TARGET
7	Equip	U113A1	APC, M113A1, TRACK
8	Equip	U113A2	APC, M113A2, TRACK
9	Equip	U113A3	APC, M113A3, TRACK
10	Equip	U113AS	APC, M113A1/M113A3, TRACK, SCUD TARGET
11	Equip	U113BM	APC, M113A1, TRACK, W/25MM BUSHMASTER
12	Equip	UB113BS	APC, M113A1, TRACK, W/25MM BUSHMASTER, SCUD TARGET
13	Equip	U113TW	APC, M113A1, TRACK, W/TOW
14	Equip	U120IT	TANK, S. KOREAN K1A1, W/120MM MAIN GUN
15	Equip	U120TS	TANK, S. KOREAN K1A1, W/120MM MAIN GUN, SCUD TARGET
16	Equip	U130ML	MRLS, W/130MM TUBE
17	Equip	U577A1	CMD POST VEH, M577/A1/A2/A3
18	Equip	U577CP	CMD POST VEH, M577/A1/A2/A3
19	Equip	U577CS	CMD POST VEH, M577/A1/A2/A3, SCUD TARGET
20	Equip	UA1A10	FIX WING, A-10A, WART HOG CLOSE AIR SUPPORT
21	Equip	UA1EF	FIX WING, EF-2000 (EURO FIGHTER)
22	Equip	UA1F16	FIX WING, F-16
23	Equip	UA1JFS	FIX WING, JOINT STRIKE FIGHTER
24	Equip	UA1TOR	FIX WING; TORNADO
25	Equip	UA2A10	FIX WING, A-10A, WART HOG CLOSE AIR SUPPORT
26	Equip	UA2EF	FIX WING, EF-2000 (EURO FIGHTER)
27	Equip	UA2F16	FIX WING, F-16
28	Equip	UA2JFS	FIX WING, JOINT STRIKE FIGHTER
29	Equip	UA2TOR	FIX WING, TORNADO
30	Equip	UA3A10	FIX WING, A-10A, WART HOG CLOSE AIR SUPPORT
31	Equip	UA3F16	FIX WING, F-16
32	Equip	UA3JFS	FIX WING, JOINT STRIKE FIGHTER
33	Equip	UA3TOR	FIX WING, TORNADO
34	Equip	UA4A10	FIX WING, A-10A, WART HOG CLOSE AIR SUPPORT
35	Equip	UA4F16	FIX WING, F-16

Figure 3.2 The "Platforms" worksheet lists all platforms used by COSAGE (both shooter and target), weapon systems, munitions, platform abbreviations and descriptions.

Initial Platform Numbers

Platform Name	Initial Platform Number
RMD500	40
RHOPLT	40
RMIG27	7
RSU25	13
RAD85A	48
RAD57A	304
RAD14Z	112
RL240Z	96
RL122V	288
RG170Z	112
RG152Z	648
RG130Z	168
RG122Z	560
RG130A	432
UH58D2	7
UHCAH1	12
UHC58D	30
UHCL64	24
UHCH64	24
UACL64	48.87
UACH64	59.13
UA4F16	4
UA3F16	2
UA4A10	6
UA3A10	8
UA2A10	8
UA1A10	8
UMLRS	27
UH155Z	144
UM120Z	54
UH155A	80

Figure 3.3 The "Initial" worksheet provides Red and Blue initial platform numbers for the designated COSAGE combat posture.

d. *Indirect*

The "Indirect" worksheet provides the analytical model with a list of indirect fire shots, averaged over total COSAGE replications, for each shooter / weapon / munition 3-tuple present (see Figure 3.4).

e. *Kbr and Krb*

The "Kbr" and "Krb" worksheets provide both the user and the analytical model with Bayesian estimates for survival probabilities, as well as the estimated average kill rate for each shooter / munition / target 3-tuple present (see Figure 3.5). The worksheets also provide the model firing rates (see Section 3, equations 3.4 and 3.10) for each shooter / munition / target 3-tuple present for a 24-hour period (column I), which is required to project the model beyond the initial 48 hours modeled by COSAGE. The

worksheet “Kbr” shown in Figure 3.5 represents Blue forces killing Red forces. All information shown is identical for worksheet “Krb,” with the exception that “Krb” models Red forces killing Blue forces.

Indirect Fire Shot Totals

Shooter	Weapon	Munition	Total Indirect Shots (S_{BR} & S_{RB})
RMIG27	BOMB	R250B	118.5
RSU25	BOMB	R250B	70.87
RL240Z	ICM	240ICM	3662
RL240Z	HE	L240HE	6372.75
RL122V	HE	L122HE	17286.23
RL122V	FASCAM	RDRAAM	34770.03
RL122V	ICM	122ICM	34770.03
RG170Z	HE	G170HE	1003.14
RG152Z	HE	152RAP	2267.09
RG152Z	SMOKE	H152SM	25059.74
RG152Z	FASCAM	RDRAAM	25059.74
RG152Z	ILLUM	H152IL	25059.74
RG152Z	HE	G152HF	25059.74
RG152Z	ICM	152ICM	81478.97
RG130Z	ILLUM	130IL	47314.61
RG130Z	SMOKE	G130SM	47314.61
RG130Z	FASCAM	RDRAAM	47314.61
RG130Z	ICM	130ICM	47314.61
RG130Z	HE	G130HE	14278.24
RG122Z	HE	G122HF	55461.36
RG122Z	HE	122RAP	48065.51
RG130A	ILLUM	130IL	9857.33
RG130A	SMOKE	G130SM	9857.33
RG130A	HE	G130HE	9857.33
RG130A	FASCAM	RDRAAM	32252.38
RG130A	ICM	130ICM	32252.38
RL107A	HE	L107HE	54467.5
RM120M	SMOKE	M120SM	101584.68

Figure 3.4 The worksheet “Indirect” provides the analytical model with total indirect fire shots for each shooter/munition/target 3-tuple present.

A1 Kill Data for Blue Platforms Killing Red Targets

Kill Data for Blue Platforms Killing Red Targets

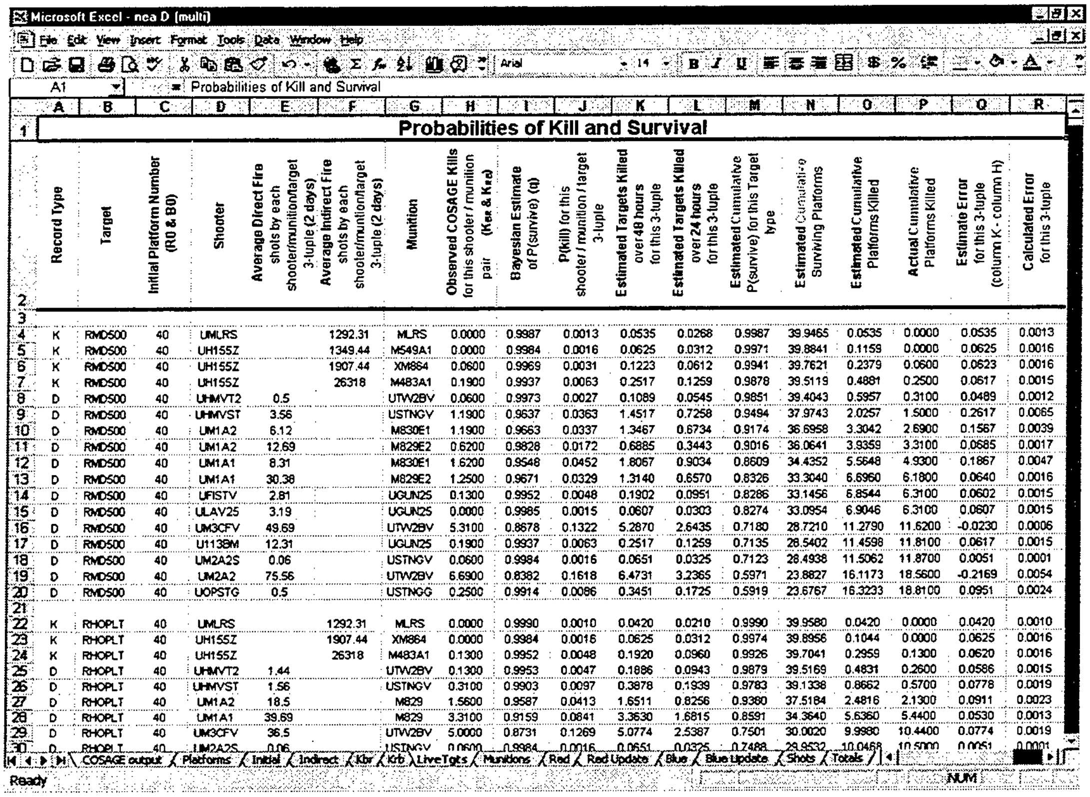
	Red target	Initial target density (R_0)	Blue shooter	Blue munition	Initial Blue shooter density (B_0)	Average Total Indirect Shots	Average shots taken by Blue shooter / munition pair against Red target	Average shots per Blue shooter / munition pair per 2 days	Average shots per Blue shooter / munition pair per day	Average Red platforms killed by Blue shooter / munition pair	Average kills on dead Red targets by Blue shooter / munition pair	Simple P_{kill} for Blue shooter / munition pair against Red target	Record Type	Bayes Estimate P(survive single shot)	Bayes Estimate P(survive all shots) 2 days (q)	Bayes Estimate P(survive all shots) 1 day ($q^{1/2}$)	Estimated Average Kill Rate (1 day) for this shooter / munition / target 3-tuple
3	RMD500	40	UMLRS	MLRS	27	1292.31	0.56	0.0207	0.0104	0	0	0.0000	K	0.9088	0.9987	0.9993	0.0044
4	RMD500	40	UH155Z	M549A1	144	1349.44		9.3711	4.6856	0		Indirect	K	1.0000	0.9984	0.9992	0.0000
5	RMD500	40	UH155Z	XM864	144	1907.44		13.2461	6.6231	0.06		Indirect	K	1.0000	0.9969	0.9985	0.0000
6	RMD500	40	UH155Z	M483A1	144	26318		182.7639	91.3819	0.19		Indirect	K	1.0000	0.9937	0.9968	0.0000
7	RMD500	40	UHMVT2	UTW2BV	44		0.5	0.0114	0.0057	0.06	0	0.1200	D	0.8040	0.9973	0.9986	0.0051
8	RMD500	40	UHMVST	USTNGV	48		3.56	0.0742	0.0371	1.19	0	0.3343	D	0.6601	0.9637	0.9817	0.2332
9	RMD500	40	UM1A2	M830E1	100		6.12	0.0612	0.0306	1.19	0	0.1944	D	0.7994	0.9663	0.9830	0.2058
10	RMD500	40	UM1A2	M829E2	100		12.69	0.1269	0.0635	0.62	0	0.0489	D	0.9467	0.9828	0.9914	0.1719
11	RMD500	40	UM1A1	M830E1	228		8.31	0.0364	0.0182	1.62	0	0.1949	D	0.8005	0.9548	0.9772	0.1899
12	RMD500	40	UM1A1	M829E2	228		30.38	0.1332	0.0666	1.25	0	0.0411	D	0.9570	0.9671	0.9834	0.3386
13	RMD500	40	UFISTV	UGUN25	40		2.81	0.0703	0.0351	0.13	0	0.0463	D	0.9344	0.9952	0.9976	0.0346
14	RMD500	40	ULAV25	UGUN25	46		3.19	0.0693	0.0347	0	0	0.0000	D	0.9811	0.9985	0.9992	0.0112
15	RMD500	40	UM3CFV	UTW2BV	108		49.69	0.4601	0.2300	5.31	0	0.1069	D	0.8922	0.8678	0.9316	2.1124
16	RMD500	40	U113BM	UGUN25	136		12.31	0.0905	0.0453	0.19	0	0.0154	D	0.9797	0.9937	0.9968	0.0530
17	RMD500	40	UM2A2S	USTNGV	19		0.06	0.0032	0.0016	0.06	0	1.0000	D	0.3378	0.9984	0.9992	0.0015
18	RMD500	40	UM2A2	UTW2BV	214		75.56	0.3531	0.1765	6.69	0	0.0885	D	0.9108	0.8382	0.9155	2.4935
19	RMD500	40	UOPSTG	USTNGG	44		0.5	0.0114	0.0057	0.25	0	0.5000	D	0.5000	0.9914	0.9957	0.0130
20	RHOPLT	40	UMLRS	MLRS	27	1292.31	0.19	0.0070	0.0035	0	0	0.0000	K	0.8016	0.9990	0.9995	0.0017
21	RHOPLT	40	UH155Z	XM864	144	1907.44		13.2461	6.6231	0		Indirect	K	1.0000	0.9984	0.9992	0.0000
22	RHOPLT	40	UH155Z	M483A1	144	26318		182.7639	91.3819	0.13		Indirect	K	1.0000	0.9952	0.9976	0.0000
23	RHOPLT	40	UHMVT2	UTW2BV	44		1.44	0.0327	0.0164	0.13	0	0.0903	D	0.8770	0.9953	0.9976	0.0200
24	RHOPLT	40	UHMVST	USTNGV	48		1.56	0.0325	0.0163	0.31	0	0.1987	D	0.7789	0.9903	0.9951	0.0384
25	RHOPLT	40	UM1A2	M829	100		18.5	0.1850	0.0925	1.56	0	0.0843	D	0.9129	0.9587	0.9791	0.4841

COSAGE output / Platforms / Initial / Indirect / Kbr / Krb / LiveTgts / Munitions

Figure 3.5 The worksheets "Kbr" and "Krb" represent survival estimates and estimated kill rates for Blue forces killing Red and Red forces killing Blue respectively.

f. LiveTgts

Worksheet "LiveTgts" pulls Bayesian estimates, described in Section 3, for survival probability from worksheets "Kbr" and "Krb," using them to calculate estimated number of targets killed for each shooter / munition / target 3-tuple present, as well as estimated cumulative number of kills for each target type present for both 48 and 24-hour periods (see Figure 3.6). Since estimating cumulative platform kills in column O is an additive process, it is important that, in order to estimate total cumulative kills, the user read the last entry before the break for each target type. For an explanation of "Calculated Error" (column R), see Equation (3.14).



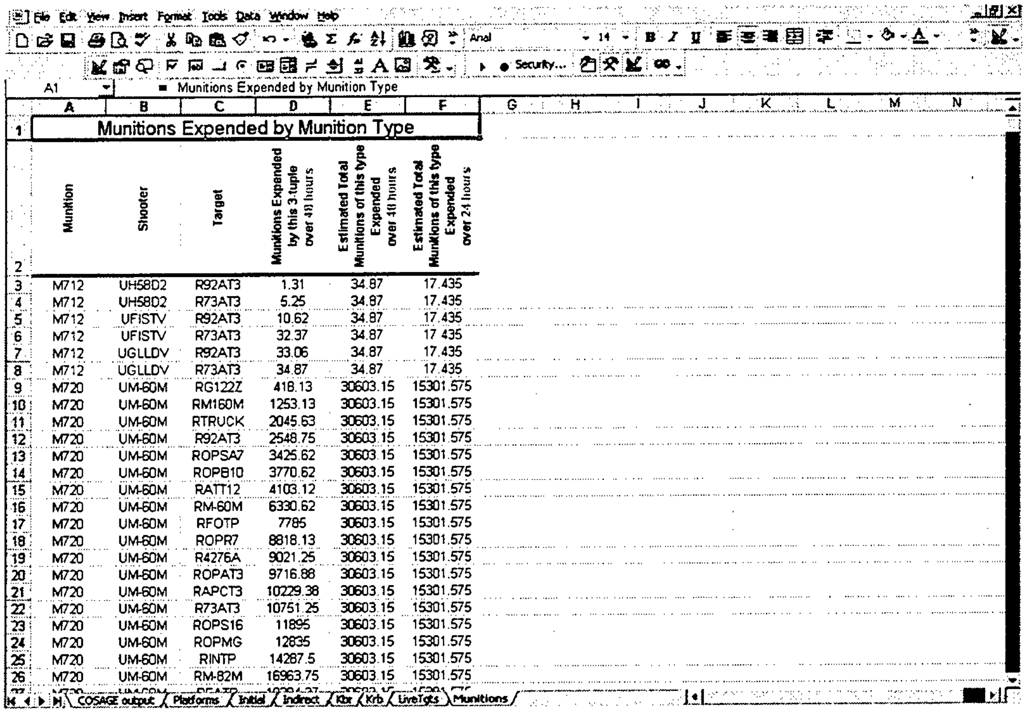
Probabilities of Kill and Survival

	Record Type	Target	Initial Platform Number (R0 & B0)	Shooter	Average Direct Fire shots by each shooter/munition/target 3-tuple (2 days)	Average Indirect Fire shots by each shooter/munition/target 3-tuple (2 days)	Munition	Observed COSAGE Kills for this shooter / munition pair (K_{BR} & K_{RB})	Bayesian Estimate of P(survive) (q)	P(kill) for this shooter / munition / target 3-tuple	Estimated Targets Killed over 48 hours for this 3-tuple	Estimated Targets Killed over 24 hours for this 3-tuple	Estimated Cumulative P(survive) for this Target type	Estimated Cumulative Surviving Platforms	Estimated Cumulative Platforms Killed	Actual Cumulative Platforms Killed	Estimate Error for this 3-tuple (column K - column H)	Calculated Error for this 3-tuple
4	K	RMD500	40	UMLRS		1292.31	MLRS	0.0000	0.9987	0.0013	0.0535	0.0268	0.9987	39.9465	0.0535	0.0000	0.0535	0.0013
5	K	RMD500	40	UH155Z		1349.44	M549A1	0.0000	0.9984	0.0016	0.0625	0.0312	0.9971	39.8841	0.1159	0.0000	0.0625	0.0016
6	K	RMD500	40	UH155Z		1907.44	XM864	0.0600	0.9969	0.0031	0.1223	0.0612	0.9941	39.7621	0.2379	0.0600	0.0623	0.0016
7	K	RMD500	40	UH155Z		26318	M483A1	0.1900	0.9937	0.0063	0.2517	0.1259	0.9878	39.5119	0.4881	0.2500	0.0617	0.0015
8	D	RMD500	40	UHMVT2	0.5		UTW2BV	0.0600	0.9973	0.0027	0.1089	0.0545	0.9851	39.4043	0.5957	0.3100	0.0489	0.0012
9	D	RMD500	40	UHMVST	3.56		USTNGV	1.1900	0.9637	0.0363	1.4517	0.7258	0.9494	37.9743	2.0257	1.5000	0.2617	0.0065
10	D	RMD500	40	UM1A2	6.12		M830E1	1.1900	0.9663	0.0337	1.3467	0.6734	0.9174	36.6958	3.3042	2.6900	0.1567	0.0039
11	D	RMD500	40	UM1A2	12.69		M829E2	0.6200	0.9828	0.0172	0.6885	0.3443	0.9016	36.0641	3.9359	3.3100	0.0685	0.0017
12	D	RMD500	40	UM1A1	8.31		M830E1	1.6200	0.9548	0.0452	1.8067	0.9034	0.8609	34.4352	5.5648	4.9300	0.1867	0.0047
13	D	RMD500	40	UM1A1	30.38		M829E2	1.2500	0.9671	0.0329	1.3140	0.6570	0.8326	33.3040	6.6960	6.1800	0.0640	0.0016
14	D	RMD500	40	UFISTV	2.81		UGUN25	0.1300	0.9952	0.0048	0.1902	0.0951	0.8286	33.1456	6.8544	6.3100	0.0602	0.0015
15	D	RMD500	40	ULAV25	3.19		UGUN25	0.0000	0.9985	0.0015	0.0607	0.0303	0.8274	33.0954	6.9046	6.3100	0.0607	0.0015
16	D	RMD500	40	UM3CFV	49.69		UTW2BV	5.3100	0.8678	0.1322	5.2870	2.6435	0.7180	28.7210	11.2790	11.6200	-0.0230	0.0006
17	D	RMD500	40	U113BM	12.31		UGUN25	0.1900	0.9937	0.0063	0.2517	0.1259	0.7135	28.5402	11.4598	11.8100	0.0617	0.0015
18	D	RMD500	40	UM2A2S	0.06		USTNGV	0.0600	0.9984	0.0016	0.0651	0.0325	0.7123	28.4938	11.5062	11.8700	0.0051	0.0001
19	D	RMD500	40	UM2A2	75.56		UTW2BV	6.6900	0.8382	0.1618	6.4731	3.2365	0.5971	23.8827	16.1173	18.5600	-0.2169	0.0054
20	D	RMD500	40	UOPSTG	0.5		USTNGG	0.2500	0.9914	0.0086	0.3451	0.1725	0.5919	23.6767	16.3233	18.8100	0.0951	0.0024
21																		
22	K	RHOPLT	40	UMLRS		1292.31	MLRS	0.0000	0.9990	0.0010	0.0420	0.0210	0.9990	39.9580	0.0420	0.0000	0.0420	0.0010
23	K	RHOPLT	40	UH155Z		1907.44	XM864	0.0000	0.9984	0.0016	0.0625	0.0312	0.9974	39.8956	0.1044	0.0000	0.0625	0.0016
24	K	RHOPLT	40	UH155Z		26318	M483A1	0.1300	0.9952	0.0048	0.1920	0.0960	0.9926	39.7041	0.2959	0.1300	0.0620	0.0016
25	D	RHOPLT	40	UHMVT2	1.44		UTW2BV	0.1300	0.9953	0.0047	0.1886	0.0943	0.9879	39.5169	0.4831	0.2600	0.0586	0.0015
26	D	RHOPLT	40	UHMVST	1.56		USTNGV	0.3100	0.9903	0.0097	0.3878	0.1939	0.9783	39.1338	0.8662	0.5700	0.0778	0.0019
27	D	RHOPLT	40	UM1A2	18.5		M829	1.5600	0.9587	0.0413	1.6511	0.8256	0.9380	37.5184	2.4816	2.1300	0.0911	0.0023
28	D	RHOPLT	40	UM1A1	39.69		M829	3.3100	0.9159	0.0841	3.3630	1.6815	0.8591	34.3640	5.6360	5.4400	0.0530	0.0013
29	D	RHOPLT	40	UM3CFV	36.5		UTW2BV	5.0000	0.8731	0.1269	5.0774	2.5387	0.7501	30.0020	9.9980	10.4400	0.0774	0.0019
30	D	RHOPLT	40	UM2A2S	0.06		USTNGV	0.0600	0.9984	0.0016	0.0651	0.0325	0.7488	29.9532	10.0468	10.5000	0.0051	0.0001

Figure 3.6 The "LiveTgts" worksheet displays estimated kills for each shooter / munition / target 3-tuple present, as well as estimated kills by target type. For an explanation of "Calculated Error" (column R), see Equation (3.14).

g. Munitions

"Munitions" provides the user estimated total munitions expended over a 24-hour period by munition type, as well as total munitions expended for each shooter / munition / target 3-tuple present (see Figure 3.7). Calculations for these estimates can be found in Section 3,



Munitions Expended by Munition Type

Munition	Shooter	Target	Munitions Expended by this 3-tuple over 48 hours	Estimated Total Munitions of this type Expended over 48 hours	Estimated Total Munitions of this type Expended over 24 hours
M712	UH58D2	R92AT3	1.31	34.87	17.435
M712	UH58D2	R73AT3	5.25	34.87	17.435
M712	UFISTV	R92AT3	10.62	34.87	17.435
M712	UFISTV	R73AT3	32.37	34.87	17 435
M712	UGLLDV	R92AT3	33.06	34.87	17.435
M712	UGLLDV	R73AT3	34.87	34.87	17.435
M720	UM-60M	RG122Z	418.13	30603.15	15301.575
M720	UM-60M	RM160M	1253.13	30603.15	15301.575
M720	UM-60M	RTRUCK	2045.63	30603.15	15301.575
M720	UM-60M	R92AT3	2548.75	30603.15	15301.575
M720	UM-60M	ROPSA7	3425.62	30603.15	15301.575
M720	UM-60M	ROPB10	3770.62	30603.15	15301.575
M720	UM-60M	RATT12	4103.12	30603.15	15301.575
M720	UM-60M	RM-60M	6330.62	30603.15	15301.575
M720	UM-60M	RFOTP	7785	30603.15	15301.575
M720	UM-60M	ROPR7	8818.13	30603.15	15301.575
M720	UM-60M	R4276A	9021.25	30603.15	15301.575
M720	UM-60M	ROPAT3	9716.88	30603.15	15301.575
M720	UM-60M	RAPCT3	10229.38	30603.15	15301.575
M720	UM-60M	R73AT3	10751.25	30603.15	15301.575
M720	UM-60M	ROPS16	11895	30603.15	15301.575
M720	UM-60M	ROPMG	12835	30603.15	15301.575
M720	UM-60M	RINTP	14287.5	30603.15	15301.575
M720	UM-60M	RM-82M	16963.75	30603.15	15301.575

Figure 3.7 Worksheet "Munitions" provides estimated total munitions expenditures for each shooter / munition / target 3-tuple present, as well as estimated cumulative munitions expenditures by munition type.

3. Preliminary Calculations

This section introduces the reader to some of the key calculations implemented by the analytical model. An input into the analytical model, the Bayesian estimates for survival probability, were introduced by Professor Gaver and Professor Jacobs in their paper *Discrete-Time Analytical Models for Use in Combat Systems Studies that Augment Simulation Models ("COSAGE")* (Gaver and Jacobs, 1999) in order to avoid over-emphasis on literally zero platform kills by COSAGE (a result of random sampling). Here the prior probability distribution has been *assumed to be uniform(0, 1).* Improvements can be made using actual individual shooter / weapon / target data. This is left for future work.

Estimators

Direct Fire

The Bayesian estimates for direct fire survival and kill probabilities are obtained for each combat posture, and are formulated on the assumption that each direct fire shot fired by a particular shooter is aimed at only one target. The direct fire assumption in DTAM is that, for every shooter / munition / target 3-tuple, there is a miss probability q such that q^x is the probability of surviving x shots. In other words, shots are assumed to be independent. The parameter q must be estimated from COSAGE output, but there is a danger that the estimate may be near the extreme values of 0 or 1 because of small statistical samples. We therefore adopt a Bayesian point of view where q is initially supposed to uniformly random in the interval [0, 1]. The estimate $\hat{q}$ is then taken to be the mean of the posterior distribution (Gaver and Jacobs, 1999). This point of view is responsible for the terms $1/Reps$ and $2/Reps$ in the estimates specified below; the effect is to move estimates away from extreme values toward the mean of the distribution (½). Results of these estimates can be seen in worksheets "Kbr," "Krb," and "LiveTgts."

The Bayesian estimate for the single shot survival probability for a Red target of type j_R being fired on by a Blue shooter of type j_B firing munition w_B for the initial COSAGE 48-hour period is

$$\hat{q}(j_B, w_B, j_R) = \frac{\overline{N}_{BR}(j_B, w_B, j_R, 0) - \overline{K}_{BR}(j_B, w_B, j_R, 0) + (1/Reps)}{\overline{N}_{BR}(j_B, w_B, j_R, 0) + (2/Reps)} \quad (3.1)$$

The estimate for the initial 24-hour survival probability of one Red target of type j_R against *all* shots by a given Blue shooter / munition pair (j_B, w_B) is

$$\hat{q}_{24}(j_B, w_B, j_R) = \left(\hat{q}(j_B, w_B, j_R)\right)^{(1/2)\left(\overline{N}_{BR}(j_B, w_B, j_R, 0) / R(j_R, 0)\right)} \quad (3.2)$$

where the exponent ½ scales results from 48 to 24 hours

The estimate for the average number of shots per Blue shooter of type j_B firing direct fire munition w_B at Red targets of type j_R over any 24-hour period is

$$\rho_{BR}(j_B, w_B, j_R) = \frac{1}{2}\left(\frac{\overline{N}_{BR}(j_B, w_B, j_R, 0)}{B(j_B, 0)}\right) \tag{3.3}$$

The estimate for the average rate of kill of Red targets of type j_R by each Blue shooter of type j_B firing munition w_B at Red target of type j_R over any 24-hour period is estimated as

$$\delta(j_B, w_B, j_R) = \rho_{BR}(j_B, w_B, j_R)\left(1 - \hat{q}(j_B, w_B, j_R)\right) \tag{3.4}$$

The estimate for the expected number of kills of Red targets of type j_R by all Blue shooters of type j_B firing munition w_B in a 24-hour period starting at time t is

$$R(j_R, t) - R(j_R, t+1) = \left\{1 - \hat{q}(j_B, w_B, j_R)^{\left(\frac{\rho_{BR}(j_B, w_B, j_R)\cdot B(j_B, t)}{R(j_R, t)}\right)}\right\}\{R(j_R, t)\} \tag{3.5}$$

Indirect (or Area) Fire

Unlike direct fire, indirect fire does not make the assumption that each shot fired by a given shooter / munition pair is directed at a single target. One indirect fire shot can kill more than one target if multiple targets are within its lethal area footprint. The COSAGE Killer-Victim scoreboard introduces the notion of "K" shots, which indicate the indirect fire shots "perceived" by intended targets. The implementation of the analytical model does not address "K" shots ("K" records in COSAGE output), utilizing "S" records instead to account for actual indirect fire shots fired by Blue and Red forces.

A Bayesian estimate for the survival probability for one Red platform of type j_R against a single shot by a shooter of type j_B firing munition w_B, using the initial 48-hour COSAGE output, is

$$\hat{q}\left(j_B, w_B, j_R\right) = 1 - \frac{\overline{K}_{BR}\left(j_B, w_B, j_R, 0\right) + \left(1/Reps\right)}{\overline{S}_{BR}\left(j_B, w_B, 0\right) \cdot R(j_R, 0) + \left(2/Reps\right)} \tag{3.6}$$

A Bayesian estimate for the initial 24-hour survival probability for a Red platform j_R against (½) $\overline{S}_{BR}\left(j_B, w_B, 0\right)$ shots is

$$\hat{q}_{24}\left(j_B, w_B, j_R\right) = \left[\hat{q}\left(j_B, w_B, j_R\right)\right]^{\left(\frac{1}{2}\right)\overline{S}_{BR}\left(j_B, w_B, 0\right)} \tag{3.7}$$

where the exponent ½ scales results from 48 to 24 hours

The estimated rate at which each Blue shooter of type j_B fires indirect fire munitions of type w_B at Red targets of type j_R over any 24-hour period is

$$\rho_{BR}\left(j_B, w_B, j_R\right) = \frac{1}{2}\left(\frac{\overline{S}_{BR}\left(j_B, w_B, 0\right)}{B\left(j_B, 0\right)}\right) \tag{3.8}$$

The estimate for the average rate of kill of Red targets of type j_R by each Blue shooter of type j_B firing indirect fire munition w_B over any 24-hour period is

$$\delta\left(j_B, w_B, j_R\right) = \rho\left(j_B, w_B, j_R\right)\left(1 - \hat{q}\left(j_B, w_B, j_R\right)\right) \tag{3.9}$$

The Bayesian estimate for the expected number of indirect kills of Red targets of type j_R by all Blue shooters of type j_B in 24 hours starting at time t is

$$R\left(j_R, t\right) - R\left(j_R, t+1\right) = \left\{1 - \hat{q}\left(j_B, w_B, j_R\right)^{\rho_{BR}\left(j_B, w_B, j_R\right) \cdot B\left(j_B, t\right)}\right\}\left\{R(j_R, t)\right\} \tag{3.10}$$

Estimated Cumulative Probability of Survival

The estimator for the cumulative expected survival probability against direct *or* indirect fire for target j_R against all shots from all shooter / weapon pairs present firing at that target for any 24-hour period starting at time t is

Direct Fire

$$\prod_{j_B}\prod_{w_B} \left(\hat{q}(j_B, w_B, j_R)\right)^{\rho_{BR}(j_B, w_B, j_R)\cdot B(j_B, t)/R(j_R, t)}, \forall j_R, t \tag{3.11}$$

Indirect Fire

$$\prod_{j_B}\prod_{w_B} \left(\hat{q}(j_B, w_B, j_R)\right)^{\rho_{BR}(j_B, w_B, j_R)\cdot B(j_B, t)}, \forall j_R, t \tag{3.12}$$

It is important to note that the estimators described above are shown for illustrative purposes only. Since targets must survive shots from both direct and indirect fire, the implementation of the analytical model uses the product of Equations 3.11 and 3.12 to describe the cumulative survival probability against both direct and indirect fire.

The estimator for the cumulative expected survival probability for Red target j_R against all shots from all Blue shooter / munition pairs (j_B, w_B) present for any 24-hour period starting at time t is

$$\hat{q}_{surv}(j_B, j_R, t) = \text{Formula}(3.11)\cdot\text{Formula}(3.12), \forall j_R, t \tag{3.13}$$

Computing Percent Error for Estimated Kills

When implementing the Bayesian estimates, one measure of effectiveness used to evaluate the software's plausibility is the percent error in attrition calculations produced by using the Bayesian estimates when compared to the initial COSAGE 48-hour period.

The estimate for the error in attrition of Red targets of type j_R being attrited by Blue targets of type j_B firing munition w_B is

$$e = \frac{\left| R(j_R, 0)\left(1 - \hat{q}_{surv}(j_B, w_B, j_R, 0)^2\right) - \overline{K}_{BR}(j_B, w_B, j_R, 0) \right|}{R(j_R, 0)} \tag{3.14}$$

where the exponent 2 scales results for $\hat{q}_{surv}(j_B, w_B, j_R, 0)$ from 24 to 48 hours.

There is, of course, a corresponding formula for Blue attrition.

IV. THE MULTIPLE-PERIOD MODEL

A. DISCUSSION

Like the single-period analytical model, the multiple-period model is a discrete-time formulation based on Bayesian estimators of survival probabilities formulated by Professors Gaver and Jacobs. The multiple-period model utilizes the same Bayesian survival probability estimates implemented in the single-period model for both direct and indirect fire cases. All state variables introduced in the single-period model carry over to the multiple period model, with the addition of an active time subscript *t*. Unlike the single-period model, in which the subscript *t* set to zero indicates the initial 48-hour timeframe, the multiple-period model uses *t* as a 24-hour period (this can be altered at will, e.g., to 12-hour periods). The Bayesian 24-hour survival estimates and shooting rates calculated in the single period model for use in the multiple-period model are utilized to produce the following:

1. Estimated platforms surviving after the user-defined 24-hour period for each shooter / munition / target 3-tuple present for the user-specified combat posture.

2. Estimated munitions expenditures for the user-defined 24-hour period for each shooter / munition / target 3-tuple present for the user-specified combat posture.

3. Expected total munitions expended by munition type for the user-defined 24-hour period and combat posture.

The Visual Basic macros implemented in the multiple-period model require approximately six minutes to produce the output described above for each theater / posture combination for each 24-hour period. A sample of the Visual Basic formulation for the multiple-period model is included as Appendix C.

B. MULTIPLE-PERIOD MODEL EXPLANATION

As stated above, the complete implementation of the multiple-period model formulation, created by Professors Gaver and Jacobs is included as Appendix C. The following introduces the multiple-period model user to the format of the model as it is implemented in Excel workbooks.

1. Excel Workbook Format

The multiple-period model for each of the seven combat postures, for both NEA and SWA, is implemented in an expanded version of the Excel workbook utilized in the single-period model. Each workbook in the multiple-period model adds the following six worksheets to those for the single-period model described in Chapter III. The six worksheets, which are explained in the next section, are:

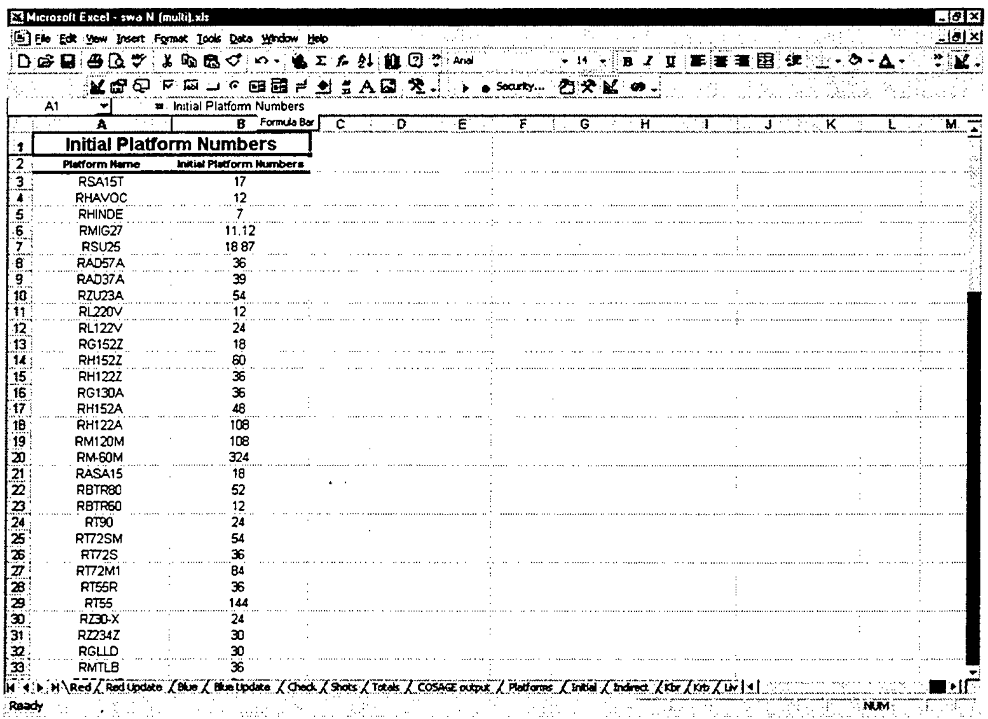
1. Red
2. Blue
3. Red Update
4. Blue Update
5. Shots
6. Totals

2. Excel Worksheets

This section is designed to familiarize the multiple-period model user with each of the six worksheets added to the Excel workbook for each combat posture, including a brief description of the key fields used in the analytical model. For illustrative purposes, all worksheets shown in this section are from combat posture SWA "N" (Blue Attack vs. Red Hasty Defense).

a. *Red and Blue*

The first two workbooks, "Red" and "Blue" provide the analytical model implementation the initial numbers of each platform present at time $t = 0$ (see Figure 4.1). Both "Red" and "Blue" worksheets are populated by running the Excel CosageUpdate macro, in which all initial numbers are drawn from the COSAGE Killer-Victim Scoreboard and apportioned between Red and Blue platforms to reduce looping times when the model is implemented.



Initial Platform Numbers	
Platform Name	Initial Platform Numbers
RSA15T	17
RHAVOC	12
RHINDE	7
RMIG27	11.12
RSU25	18.87
RAD57A	36
RAD37A	39
RZU23A	54
RL220V	12
RL122V	24
RG152Z	18
RH152Z	60
RH122Z	36
RG130A	36
RH152A	48
RH122A	108
RM120M	108
RM-60M	324
RASA15	18
RBTR80	52
RBTR60	12
RT90	24
RT72SM	54
RT72S	36
RT72M1	84
RT55R	36
RT55	144
RZ30-X	24
RZ234Z	30
RGLLD	30
RMTLB	36

Figure 4.1 The "Red" and "Blue" worksheets provide the multiple-period model with initial numbers of each platform at time $t = 0$.

b. *Red Update*

This worksheet, in conjunction with the worksheet "Blue Update," provides the user a simple model interface by which to control the number of 24-hour

periods to be modeled. Beginning with the information concerning platforms and their initial numbers provided in worksheet "Red," the "Red Update" worksheet calculates estimates of surviving Red platforms for successive 24-hour periods as a function of remaining Blue platforms, shots and kills using the Bayesian survival estimates introduced in Chapter III, Section E. In addition, the "Red Update" worksheet provides a worksheet-based interface which allows the user to simply designate how many 24-hour periods to model, for both Red and Blue forces, by entering an integer value in the dialog box and clicking the command bar (see Figure 4.2).

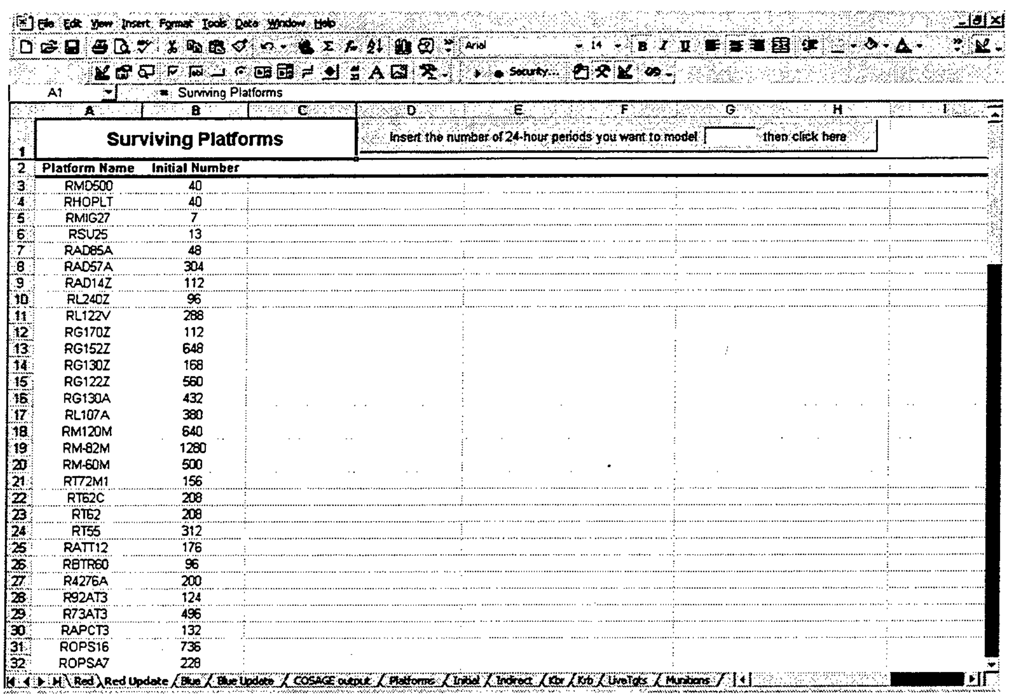


Figure 4.2 The "Red Update" worksheet includes a command bar, which allows the user to dictate how many consecutive 24-hour combat periods to model.

Once the number of periods is designated and the command button clicked, the model calculates surviving Red platforms and then surviving Blue platforms starting at time

$t = 0$, displaying successive 24-hour periods from left to right in the "Red Update" and "Blue Update" worksheets (see Figure 4.3).

c. *Blue Update*

The "Blue Update" worksheet is similar to the "Red Update" worksheet, but without the user-interface command bar. This worksheet implements the Bayesian estimates introduced in Chapter III to calculate estimates of surviving Blue platforms at 24-hour intervals, as a function of surviving Red platforms, shooting rates and kills.

Surviving Platforms — Insert the number of 24-hour periods you want to model 7 then click here

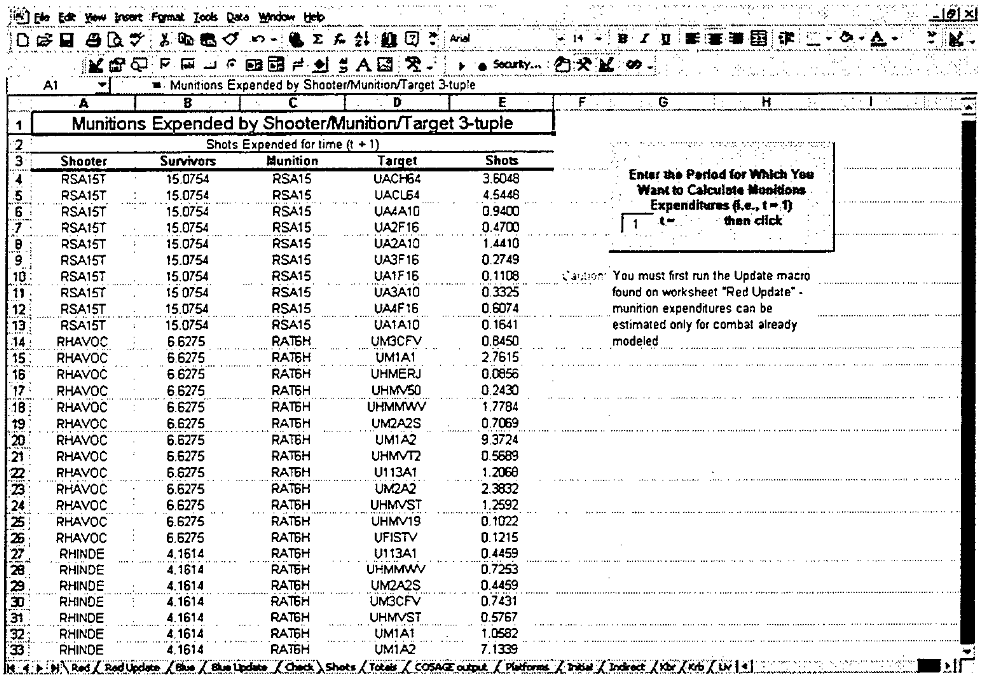
Platform Name	Initial Number	Time (t + 1)	Time (t + 2)	Time (t + 3)	Time (t + 4)	Time (t + 5)	Time (t + 6)	Time (t + 7)
RMD500	40	30.7745	23.1648	16.8206	11.5039	7.0717	3.4963	0.9661
RHOPLT	40	31.7093	24.7938	18.9596	13.9934	9.7456	6.1294	3.1443
RMIG27	7	6.9433	6.9024	6.8727	6.8509	6.8349	6.8230	6.8142
RSU25	13	12.9103	12.8423	12.7899	12.7489	12.7164	12.6902	12.6689
RAD85A	48	39.2254	32.0474	26.0690	21.0169	16.6935	12.9522	9.6844
RAD57A	304	281.1647	261.6721	244.7531	229.8634	216.5989	204.6513	193.7818
RAD14Z	112	75.7665	46.0089	22.3612	5.9554	0.0671	2.92E-160	0.00E+00
RL240Z	96	80.3931	65.9722	52.5970	40.1948	28.7634	18.4043	9.4299
RL122V	288	247.8814	212.5081	180.6020	151.3842	124.3626	99.2328	75.8372
RG170Z	112	110.5391	109.3661	108.3990	107.5855	106.8901	106.2880	105.7609
RG152Z	648	578.9001	528.2610	489.5253	458.9306	434.1559	413.6886	396.4991
RG130Z	168	148.8114	134.6621	123.7823	115.1510	108.1353	102.3207	97.4240
RG122Z	560	520.5241	490.8195	467.6154	448.9728	433.6621	420.8625	410.0039
RG130A	432	426.4343	421.9843	418.3340	415.2805	412.6865	410.4545	408.5133
RL107A	380	376.6357	373.9650	371.7855	369.9691	368.4299	367.1078	365.9590
RM120M	640	618.7572	602.0826	588.5833	577.3970	567.9559	559.8689	552.8563
RM-82M	1280	1212.8619	1162.4060	1123.0627	1091.5208	1065.6764	1044.1233	1025.8842
RM-60M	500	470.8341	449.2208	432.5736	419.3739	408.6669	399.8207	392.3996
RT72M1	156	124.5700	98.1719	75.7765	56.6533	40.2848	26.3405	14.7164
RT62C	208	159.7188	120.0791	87.2529	59.9949	37.5019	19.4462	6.3218
RT62	208	157.0578	114.8343	79.6887	50.5790	27.0561	9.6193	0.7086
RT55	312	223.3050	151.4707	93.5717	48.1758	15.8703	0.8384	2.88E-22
RATT12	176	147.8009	124.7761	105.5991	89.3414	75.3261	63.0501	52.1460
RBTR60	96	67.2145	43.4525	23.9708	9.0624	0.8487	4.24E-11	0.00E+00
R4276A	200	179.0678	161.7927	147.2377	134.7576	123.8850	114.2720	105.6590
R92AT3	124	96.8057	74.3832	55.6623	39.9178	26.6677	15.6560	6.9505
R73AT3	496	390.2348	302.7232	229.4230	167.5408	115.1622	71.1598	35.4317
RAPCT3	132	105.2053	82.9447	64.2091	48.2932	34.6989	23.0968	13.3448
ROPS16	736	632.5729	550.3008	482.7363	425.8465	376.9386	334.1331	296.0883
ROPSA7	228	196.4813	171.5986	151.4510	134.8105	120.8283	108.8858	98.5078

Figure 4.3 On worksheet "Red Update," when the user enters an integer value in the text box and clicks the command bar, the model automatically updates estimated surviving platforms.

d. *Shots*

The "Shots" worksheet utilizes 24-hour shooting rates calculated in the single-period model and surviving platforms calculated in worksheets "Red Update" and

"Blue Update" to generate estimated shots for both direct and indirect fire weapons for each shooter / munition / target 3-tuple present. These estimates do not account for target numbers equal to zero. As a result, they tend to overstate munitions expenditures; this can be corrected by introducing an indicator variable that goes to zero with target number. Re-allocation of weapons will be treated in later work. Included on the worksheet is an interface that allows the user, once the Update model is completed, to estimate munitions expenditures for a specific 24-hour period (see Figure 4.5). Once the desired 24-hour period is entered and the command button clicked, the multiple-period model populates both the "Shots" and "Totals" worksheets. The current assumption made in Chapter II, Section B, that shooting rate per Blue shooter is independent of target number, allows a somewhat naive approach to attrition calculations for successive 24-hour periods without sacrificing the plausibility of the estimates for direct and indirect fire attrition. Sample multiple-period results are included in Chapter V.



Munitions Expended by Shooter/Munition/Target 3-tuple

Shots Expended for time (t + 1)

Shooter	Survivors	Munition	Target	Shots
RSA15T	15.0754	RSA15	UACH64	3.6048
RSA15T	15.0754	RSA15	UACL64	4.5448
RSA15T	15.0754	RSA15	UA4A10	0.9400
RSA15T	15.0754	RSA15	UA2F16	0.4700
RSA15T	15.0754	RSA15	UA2A10	1.4410
RSA15T	15.0754	RSA15	UA3F16	0.2749
RSA15T	15.0754	RSA15	UA1F16	0.1108
RSA15T	15.0754	RSA15	UA3A10	0.3325
RSA15T	15.0754	RSA15	UA4F16	0.6074
RSA15T	15.0754	RSA15	UA1A10	0.1641
RHAVOC	6.6275	RAT6H	UM3CFV	0.8450
RHAVOC	6.6275	RAT6H	UM1A1	2.7615
RHAVOC	6.6275	RAT6H	UHMERJ	0.0856
RHAVOC	6.6275	RAT6H	UHMV50	0.2430
RHAVOC	6.6275	RAT6H	UHMMWV	1.7784
RHAVOC	6.6275	RAT6H	UM2A2S	0.7069
RHAVOC	6.6275	RAT6H	UM1A2	9.3724
RHAVOC	6.6275	RAT6H	UHMVT2	0.5689
RHAVOC	6.6275	RAT6H	U113A1	1.2068
RHAVOC	6.6275	RAT6H	UM2A2	2.3832
RHAVOC	6.6275	RAT6H	UHMVST	1.2592
RHAVOC	6.6275	RAT6H	UHMV19	0.1022
RHAVOC	6.6275	RAT6H	UFISTV	0.1215
RHINDE	4.1614	RAT6H	U113A1	0.4459
RHINDE	4.1614	RAT6H	UHMMWV	0.7253
RHINDE	4.1614	RAT6H	UM2A2S	0.4459
RHINDE	4.1614	RAT6H	UM3CFV	0.7431
RHINDE	4.1614	RAT6H	UHMVST	0.5767
RHINDE	4.1614	RAT6H	UM1A1	1.0582
RHINDE	4.1614	RAT6H	UM1A2	7.1339

Figure 4.5 The "Shots" worksheet provides a simple interface by which the user can predict munitions expenditures for a specified 24-hour period.

e. *Totals*

As stated above, when the user-interface on the "Shots" worksheet is activated, the "Totals" worksheet is populated with shot data from the desired 24-hour combat period. The "Totals" worksheet then calculates total munitions expenditures, both by shooter / munition / target 3-tuple, as seen in worksheet "Shots," and as total expenditures for the user-defined 24-hour period by munition type in the "Totals" worksheet (see Figure 4.6). As expected, when compared to the single-period model, the total numbers of munitions fired by all shooters decreases in successive 24-hour periods as shooters are attrited, even though, as discussed above, shooting rate per shooter remains constant throughout the combat posture. As noted before, the calculations do not

reflect the numbers of targets present. Hence, the calculations tend to overstate the munitions expended. This will be corrected in later work.

Total Munitions Expended by Type

Munitions Expended for time (t + 1)

Munition	Shooter	Target	Cumulative Shots	Total Shots of This Munition
122ICM	RL122V	UM-TP	209.5738	7544.6579
122ICM	RL122V	UM120Z	419.1477	7544.6579
122ICM	RL122V	UH105A	628.7215	7544.6579
122ICM	RL122V	UTRUCK	838.2953	7544.6579
122ICM	RL122V	UFAFO	1047.8692	7544.6579
122ICM	RL122V	UOPSAW	1257.4430	7544.6579
122ICM	RL122V	UOP203	1467.0168	7544.6579
122ICM	RL122V	UOPAT4	1676.5906	7544.6579
122ICM	RL122V	UOPDRG	1886.1645	7544.6579
122ICM	RL122V	UTPQ36	2095.7383	7544.6579
122ICM	RL122V	UFDCVH	2305.3121	7544.6579
122ICM	RL122V	UOP60G	2514.8860	7544.6579
122ICM	RL122V	UHMV50	2724.4598	7544.6579
122ICM	RL122V	UM1A2	2934.0336	7544.6579
122ICM	RL122V	UHMV19	3143.6075	7544.6579
122ICM	RL122V	UINTP	3353.1813	7544.6579
122ICM	RL122V	UM2A2	3562.7551	7544.6579
122ICM	RL122V	UHMVST	3772.3289	7544.6579
122ICM	RL122V	UPPS15	3981.9028	7544.6579
122ICM	RL122V	U113A1	4191.4766	7544.6579
122ICM	RL122V	UHMMWV	4401.0504	7544.6579
122ICM	RL122V	UMLRS	4610.6243	7544.6579
122ICM	RL122V	UFATP	4820.1981	7544.6579
122ICM	RL122V	UH155Z	5029.7719	7544.6579
122ICM	RL122V	UH155A	5239.3458	7544.6579
122ICM	RL122V	UM-60M	5448.9196	7544.6579
122ICM	RL122V	UM2A2S	5658.4934	7544.6579
122ICM	RL122V	UFISTV	5868.0673	7544.6579
122ICM	RL122V	UOPSTG	6077.6411	7544.6579

Figure 4.6 The "Totals" worksheet provides munitions expenditures both by shooter / munition / target 3-tuple, and as totals for each munition type.

C. CHANGING COMBAT POSTURES

1. Discussion

The implementation of the multiple-period model introduces the user to the added benefit of being able to switch from one combat posture to another after each 24-hour period within the same theater. As those who have studied history know, entire wars often shift from one combat posture to another as the balance of power shifts, rarely remaining in one combat posture for more than a few days at a time. COSAGE provides the user a total of seven combat postures, which are modeled in both NEA and SWA. As implemented by many combat simulations, including CEM, battles are often represented

by a "piston" model, which is forced back and forth as the balance of power shifts from Blue to Red and vice versa (CAA, 1995). By introducing the concept of shifting combat postures, the multiple-period model may more closely predict platform survival over the entire combat duration.

2. Model Implementation

The implementation of the multiple-period model is capable of changing combat postures after each 24-hour combat period. Although the model does not include a worksheet-based interface, the procedure for switching combat postures involves minimal user input, and can be accomplished in under one minute. Additionally, because the implementation is based on COSAGE, in which all postures in a given theater share a common platform set, albeit with different initial numbers, the user need not concern himself with the possibility of data "gaps." The procedure for switching from one combat posture to another is as follows:

a. Modeling the First 24 Hours

The user dictates the posture in which combat is to begin, or in which posture combat starts for a user-defined phase. Once the posture is chosen, simply run the multiple-period model for a single 24-hour period using the interface on worksheet "Red Update." At the user's discretion, the initial posture may be run for more than a single 24-hour period.

b. Formatting the Output

Once the formulation has completed the desired iterations of platform survival estimates, the output on worksheets "Red Update" and "Blue Update" requires

some quick formatting. COSAGE dictates that each posture for a given theater (NEA or SWA) begin at time $t = 0$ with the same types of platform, although many begin with different initial numbers (CAA, 1993). As a result, platform lists and the calculations associated with those lists, are interchangeable. In order to ensure that platforms simulated in the analytical model will match on a case by case basis from posture to posture, the platforms must be placed in alphabetical order. Once the desired number of 24-hour iterations is complete, the user need only alphabetize the results by selecting entire rows of data in the Excel worksheets "Red Update" and "Blue Update" (see Figure 4.7).

Once the rows containing all platform names, initial numbers and estimated survivors have been highlighted, the user need only click the Sort Ascending button on the Excel toolbar. After repeating the procedure on the "Red Update" and "Blue Update" (source) worksheets in the next dictated posture's (target) workbook, the data is ready for transfer and continued modeling. By copying the last 24-hour estimate column from the source worksheets and pasting them into the "Initial Number" column of the appropriate target worksheets, the user is ready to continue modeling combat. Because the multiple-period model implementation does not affect the worksheets "Red" and "Blue," initial platform numbers remain unchanged in these worksheets.

Surviving Platforms

Insert the number of 24-hour periods you want to model: 7 then click here

Platform Name	Initial Number	Time (t + 1)	Time (t + 2)	Time (t + 3)	Time (t + 4)	Time (t + 5)	Time (t + 6)	Time (t + 7)
RSA15T	17	15.0754	13.2195	11.4284	9.7004	8.0360	6.4382	4.9140
RHAVDC	12	6.6275	2.6057	0.3093	0.0000	0.0000	0.0000	0.0000
RHINDE	7	4.1614	2.0045	0.5384	0.0064	0.0000	0.0000	0.0000
RMIG27	11.12	11.0592	11.0068	10.9597	10.9159	10.8737	10.8326	10.7921
RSU25	18.87	18.6231	18.4047	18.2052	18.0175	17.8367	17.6602	17.4868
RAD57A	36	32.0354	28.2446	24.6145	21.1366	17.8065	14.6241	11.5940
RAD37A	39	28.0389	18.4157	10.1937	3.7612	0.2919	0.0000	0.0000
RZU23A	54	44.9211	36.4018	28.4119	20.9553	14.0875	7.9668	3.0024
RL220V	12	3.5343	0.0567	0.0000	0.0000	0.0000	0.0000	0.0000
RL122V	24	9.8201	1.2951	0.0000	0.0000	0.0000	0.0000	0.0000
RG152Z	18	11.0103	6.2827	3.1183	1.1080	0.1176	0.0000	0.0000
RH152Z	60	36.7975	20.8155	9.9821	3.1537	0.1747	0.0000	0.0000
RH122Z	36	24.4358	16.3040	10.6594	6.4987	3.6477	1.6956	0.4800
RG130A	36	26.2572	18.0575	11.2255	5.7192	1.7526	0.0513	0.0000
RH152A	48	37.9341	29.2875	21.8620	15.4774	10.0675	5.5942	2.1670
RH122A	108	89.8792	74.4611	61.2704	49.9449	40.2840	31.8216	24.6133
RM120M	108	100.1458	92.8339	86.9916	79.5659	73.5192	67.8217	62.4494
RM-60M	324	241.1676	178.7885	131.5306	95.5914	68.2098	47.3440	31.4271
RASA15	18	11.4935	6.0324	1.9334	0.0684	0.0000	0.0000	0.0000
RBTR80	52	30.2256	15.4777	5.9028	0.8357	0.0000	0.0000	0.0000
RBTR60	12	6.7111	3.1899	1.0016	0.0552	0.0000	0.0000	0.0000
RT90	24	13.9088	6.0392	1.0464	0.0001	0.0000	0.0000	0.0000
RT72SM	54	33.6185	17.0140	4.9680	0.0948	0.0000	0.0000	0.0000
RT72S	36	25.8302	17.3811	10.3847	4.7981	1.0342	0.0012	0.0000
RT72M1	84	55.2195	31.6534	13.2666	1.9552	0.0000	0.0000	0.0000
RT55R	36	20.9336	9.3663	1.8929	0.0013	0.0000	0.0000	0.0000
RT55	144	101.9136	67.1548	38.6024	16.2539	2.4384	0.0000	0.0000
RZ30-X	24	8.2286	0.4091	0.0000	0.0000	0.0000	0.0000	0.0000
RZ234Z	30	17.3600	7.8019	1.6668	0.0024	0.0000	0.0000	0.0000
RGLLD	30	17.1341	7.4965	1.4625	0.0008	0.0000	0.0000	0.0000

Red | Red Update | Blue | Blue Update | Check | Shots | Totals | COSAGE output | Platforms | Initial | Indirect | Kbr | Krb | Liv

Figure 4.7 By selecting the rows containing all platform information, the user can then alphabetize the data using the Sort Ascending icon to properly format the data.

c. *Modeling Continued Combat*

Now that the data from the last 24-hour combat period has been imported into the worksheets associated with the next user-dictated posture, it is ready for further use. Since shooting rates and kill probabilities are located in the "COSAGE output" worksheet of the target combat posture workbook, importing data does not in any way alter the combat characteristics of the target posture. The user may now select the number of 24-hour time steps to calculate in the new combat posture. Then, once the desired runs are complete, simply repeat the steps above to switch combat postures again.

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V. RESULTS AND CONCLUSIONS

A. RUN TYPES

As of publication, the implementation of the Discrete-Time Analytical Model (DTAM) has been run on all fourteen COSAGE summary data sets generated by Total Army Analysis 2005 (TAA05). Although many of the input parameters to COSAGE are classified, its output is not, making it an ideal candidate for further modeling exploration. DTAM should be used as a quick, unclassified tool used for estimating expected kills and munitions expenditures. However, the user must remain aware that the model is simply a tool, and that it includes some rather broad assumptions about combat modeling. While the results of the model are generally comparable to those produced during the first 48 hours of combat reported by COSAGE, there may exist outside factors unknown to the user that could make the model less accurate beyond the first 48 hours.

B. RESULTS

As shown in Appendix A, the single-period implementation of the analytical model performs well when compared to the COSAGE Killer-Victim Scoreboard, producing estimation errors in many cases well below one percent. One of the quickest and easiest ways to compare the estimates produced by the implementation of the analytical model and those produced by COSAGE is through linear regression. By regressing estimated kills on actual (observed COSAGE data) kills, the user can very quickly get a general idea of how well the implementation estimates the number of kills produced by COSAGE. To determine where potential problems lie, however, requires a different approach. While there is no best answer, one potential method to determine

goodness of fit" for each pairing of estimated versus observed COSAGE kills for the initial 48 hours of combat is formula 3.14, which is

$$error = \frac{|Estimated\ Number\ of\ Platforms\ Killed - Mean\ COSAGE\ Platforms\ Killed|}{Initial\ Number\ of\ Targets} \qquad (4.1)$$

Unlike other methods that amplify relatively small absolute errors when the observed COSAGE kills are small, this method penalizes the analytical model implementation only when the proportion of estimated targets killed differs greatly from the proportion of COSAGE observed kills for the initial 48 hours of combat. For example, for a platform with initial number 25 and observed COSAGE kills of 0.15, a kill estimate of 0.75 produces an error of 400% if $error = \frac{|Estimated\ Number\ of\ Platforms\ Killed - COSAGE\ Platforms\ Killed|}{COSAGE\ Platforms\ Killed}$. (4.2)

Expression (4.1) yields estimated kill error calculations for each platform type, displayed in Figure 5.1, which identifies potential problem cases.

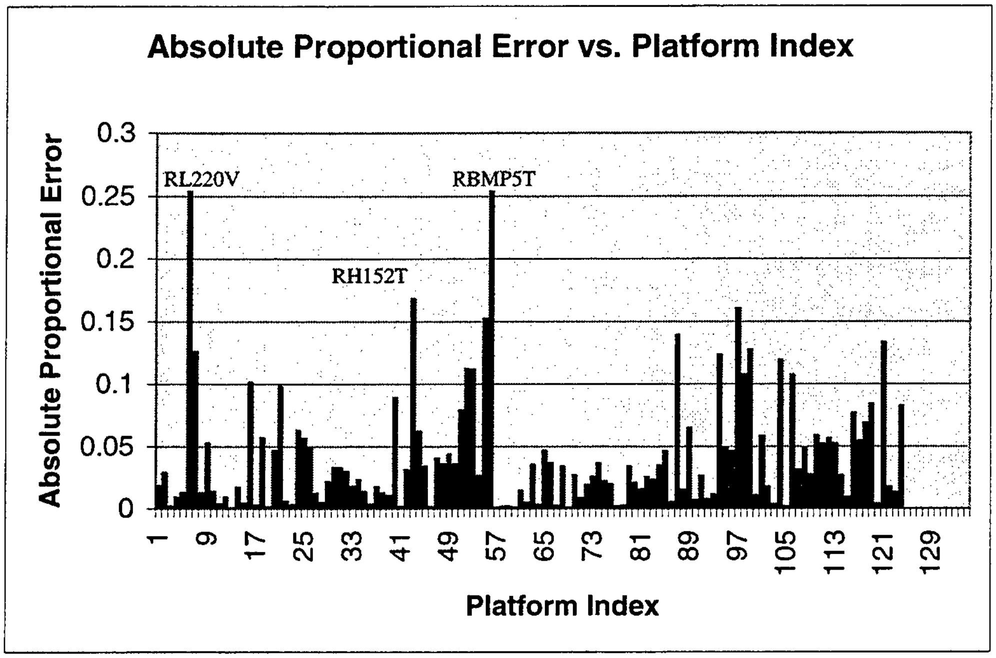


Figure 5.1 Plotting absolute proportional estimate error identifies where estimated kills produced by the analytical model implementation differ greatly from kills produced by COSAGE in the initial 48 hours of combat.

For the combat posture SWA "D" (Red Attack vs. Blue Delay), Figure 5.1 identifies three platforms for which the analytical model implementation differs significantly from COSAGE output concerning platforms killed during the initial 48 hours of combat. Table 5.1 displays these three cases in tabular form. Recall that summary data are a variable from COSAGE. Data on the number of rounds fired at each particular target are unavailable. Hence, the Bayesian estimates for survival probability include the exponent $\bar{N}_{BR}(j_B, w_B, j_R, 0) / R(j_R, 0)$ for direct fire, which assumes that each target, fired at by a weapon receives an equal allocation of all shots. In reality, each target receives only an integer number of shots, which means that some targets will receive more shots than others. For example, if a total of fifteen shots are fired at ten targets, the allocation of those fifteen shots may be explained as follows:

Let $S = 15$ (the number of shots fired)

$R = 10$ (the number of Red targets)

$\lfloor X \rfloor$ = the greatest integer in X (e.g., $\lfloor 1.7 \rfloor = 1, \lfloor 2.3 \rfloor = 2$, etc.)

Then $S = aR + b$ where b is a remainder $\left(b = S - \left\lfloor \frac{S}{R} \right\rfloor \cdot R \right)$

and $a = \left\lfloor \frac{S}{R} \right\rfloor = \left\lfloor \frac{15}{10} \right\rfloor = 1$

Hence, assuming that all targets are affected by the 15 shots, b (10) targets will receive $\left\lfloor \frac{S}{R} \right\rfloor + 1$ shots each, where 1 represents the $15 - 10 = 5$ "excess" shots, while the remaining R- b (5) targets will receive only one shot each.

Platform Name	Initial Number	COSAGE Kills	Analytical Model Implementation Kills
RH152T (152mm Howitzer)	36	15.88	37.43
RBMP5T (Armored Personnel Carrier)	84	18.38	40.12
RL220V (Multiple Rocket Launching System)	48	15.12	21.92

Table 5.1 Plotting absolute proportional error identifies potential problem cases for which estimated kill proportion differs significantly from the COSAGE kill proportion for the initial 48 hours of combat. Once potential problem cases are identified, the cause should be determnined.

Examining the kill discrepancy for platform RH152T reveals the following:

Of all shooters killing RH152T in posture SWA "D," only one, UACL64 (AH-64 helicopter) fires only one munition type at RH152T, which is the Longbow missile.

Because of its accuracy, the Longbow missile produces 5.69 kills with an average of 6.12 shots at RH152T, which represents a simple kill probability of 0.93. It appears possible that the mean COSAGE data is the result of overly-optimistic kill probability input parameters, though averaged COSAGE output does not reveal the true cause. Using Equation (3.1) to determine the single shot survival probability yields a value of 0.0789. Since there are initially 36 targets, the estimated number that survive the Longbow alone is $(36)(0.0789)^{6.12/36} = 23.37$. This corresponds to 12.63 estimated kills, whereas COSAGE produces only 5.69 kills. Because the Longbow missile, fired in small numbers, produces so many expected kills, the analytical model implementation overestimates kills of platform RH152T.

1. Implementation of the Multiple-Period Model vs. COSAGE

This section examines the implementation of the multiple-period model for combat posture NEA "F" – Blue Attack vs. Red Prepared Defense. The comparison in this section involves the multiple-period model implementation, comparing its estimated kills produced by running two consecutive 24-hour periods to the results produced by a single 48-hour period modeled by COSAGE.

To examine the plausibility of the implementation of the multiple-period model for the case NEA "F," we will look at the linear regression of estimated number of kills produced by the implementation of the analytical model for two consecutive 24-hour periods versus actual kills reported by COSAGE (see Figure 5.2).

Linear Regression

Data: NEA "F" (Blue Attack - Red Prepared Defense)
Multiple-Period Model Implementation (2 24-hour periods)

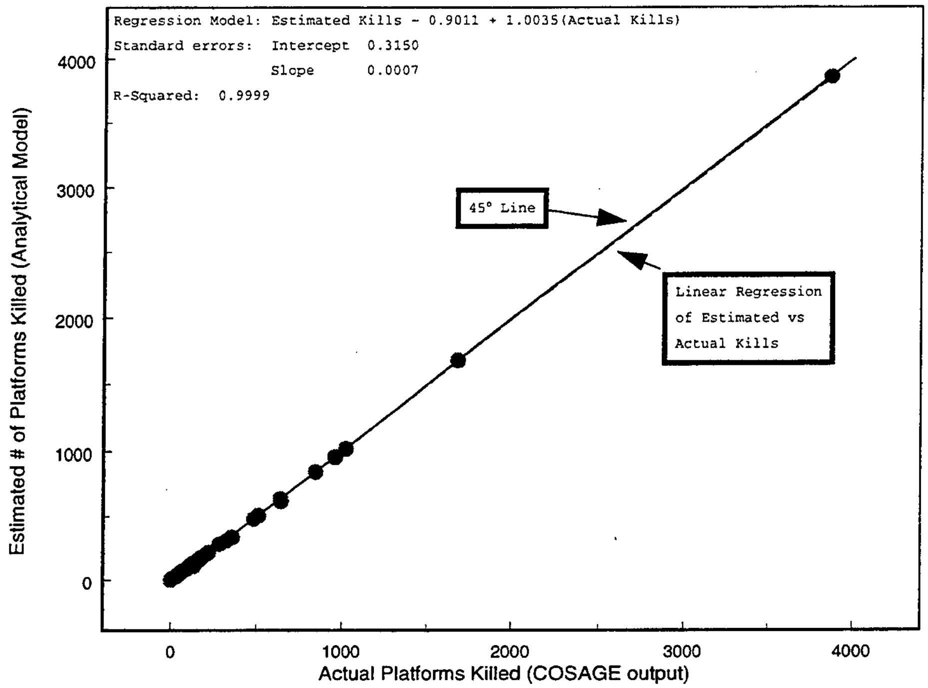


Figure 5.2 The linear regression of Estimated vs. Actual COSAGE kills confirms that the plausibility of the analytical model for estimating kills.

Based on the results shown after two consecutive 24-hour combat periods, the analytical model produces plausible results.

2. Implementation of the Multiple-Period Model vs. Implementation of the Single-Period Model

Because the single-period and multiple-period model implementations utilize the same Bayesian estimators introduced by Gaver and Jacobs, one would expect that each model implementation would produce the same estimated number of platforms killed after 48 hours of combat. In order to partially check this conjecture, we examine the linear regression of estimated multiple-period model kills versus estimated single-period model kills for the same combat posture (see Figure 5.3).

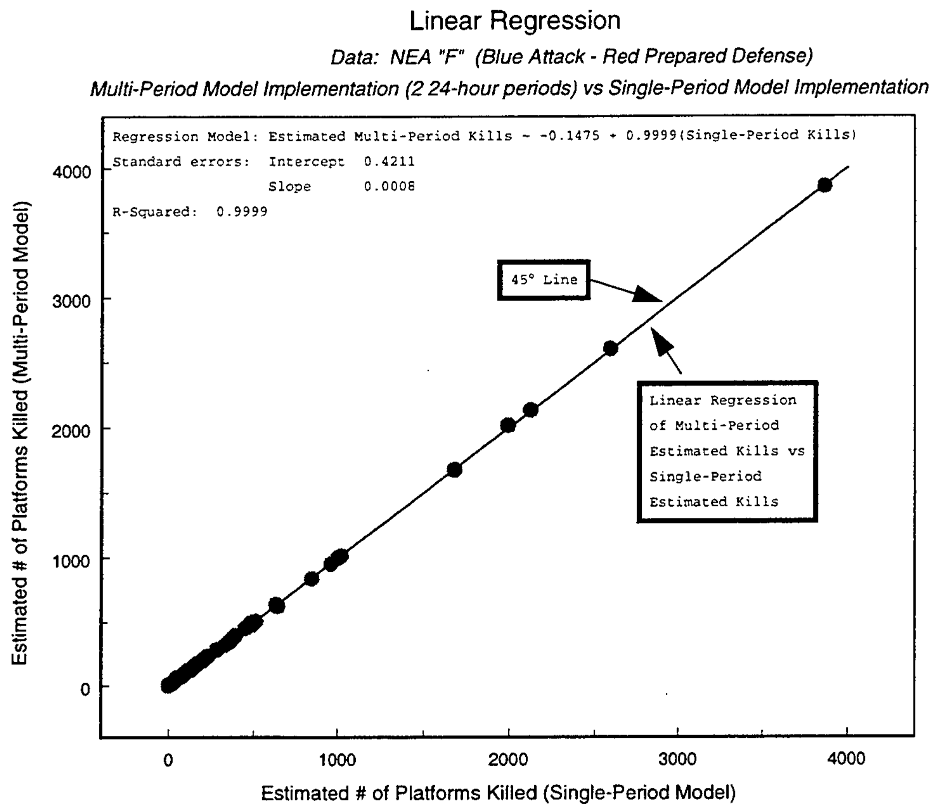


Figure 5.3 Ideally, the single-period model implementation and multiple-period model implementations should produce the same kills after 48 hours of simulated combat.

Although it appears from the plot that the single-period and multiple-period model implementations produce identical results for the estimated number of platforms killed over 48 hours of combat, the linear regression report shows otherwise. While there is measurable difference between the results from the two implementations, it is easily explained. Because the single-period model implementation models a continuous 48-hour period, it lacks the time resolution afforded by the multiple-period implementation, and as a result, calculates platform attrition only once, overlooking the attrition to shooters that will effectively diminish total estimated kills. Although its value is nearly

one, a slope value of 0.9999 may not indicate that the single-period model implementation produces the same number of platforms killed as in the multiple-period model implementation. To investigate, Figure 5.4 displays differences in the two implementations as a function of the number of targets present.

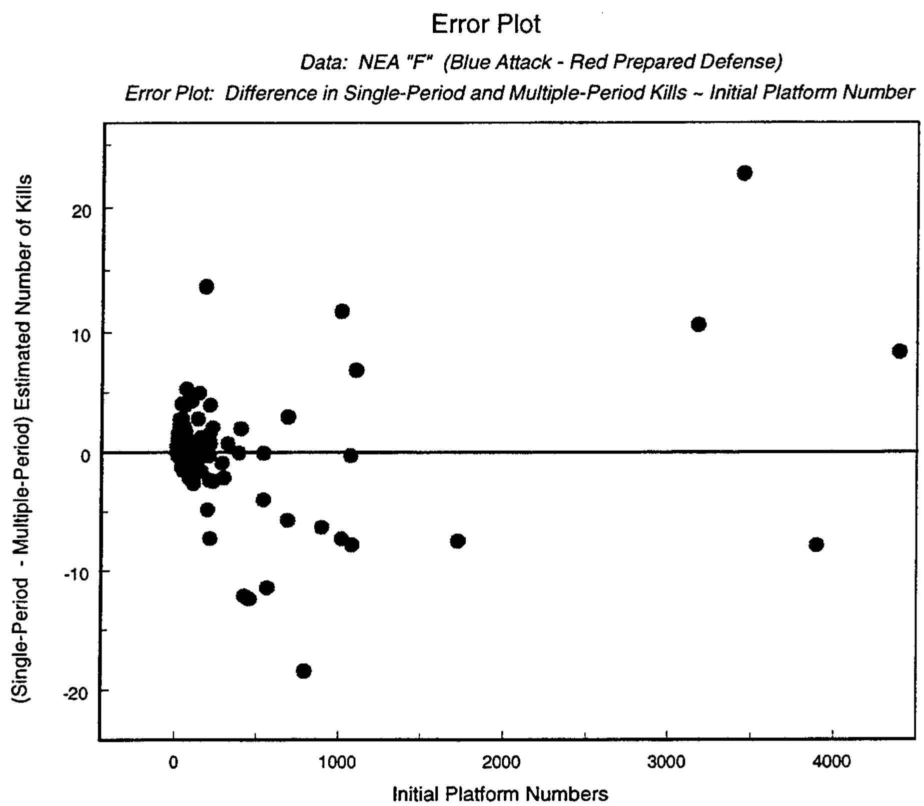


Figure 5.4 Plotting differences between the estimated kills produced by the single- and multiple-period model implementations shows that the single-period implementation does not produce a systematic bias compared to the multiple-period implementation.

3. Comparing Estimated Survivors for Different Postures

As discussed in Chapter I, COSAGE models seven different combat postures. A simple check to determine the plausibility of both the single- and multiple-period models can be accomplished by turning a *qualitative* vice a quantitative eye toward the reported

results. By recalling the different combat postures, the user should, with some sense of accuracy, be able to predict a very general outcome of the battle. For example, if the posture being modeled depicts Blue Attack vs. Red Hasty Defense (posture "N"), the user would expect to see, after several days of combat, much higher attrition rates for Red platforms than for Blue.

To provide an example of the theory outlined above, Figures 5.5 and 5.6 depict results for some platforms for seven days of combat in SWA posture "N" (Blue Attack vs. Red Hasty Defense). As expected, Blue attacking forces were able to inflict substantial attrition on Red forces fighting from a hasty defensive position. While numerous Red platforms approach zero during the first seven days of battle, the user must remember that battles rarely remain in the same combat posture for more than a few days.

Surviving Platforms

Insert the number of 24-hour periods you want to model 7 then click here

Platform Name	Initial Number	Time (t + 1)	Time (t + 2)	Time (t + 3)	Time (t + 4)	Time (t + 5)	Time (t + 6)	Time (t + 7)
RSA15T	17	15.0754	13.2195	11.4284	9.7004	8.0360	6.4382	4.9140
RHAVOC	12	6.6275	2.6057	0.3093	1.63E-08	0.00E+00	0.00E+00	0.00E+00
RHINDE	7	4.1614	2.0045	0.5384	6.37E-03	2.57E-158	0.00E+00	0.00E+00
RMIG27	11.12	11.0592	11.0068	10.9597	10.9159	10.8737	10.8326	10.7921
RSU25	18.87	18.6231	18.4047	18.2052	18.0175	17.8367	17.6602	17.4868
RAD57A	36	32.0354	28.2446	24.6145	21.1366	17.8065	14.6241	11.5940
RAD37A	39	28.0389	18.4157	10.1937	3.7612	0.2919	4.06E-15	0.00E+00
RZU23A	54	44.9211	36.4018	28.4119	20.9553	14.0875	7.9668	3.0024
RL220V	12	3.5343	0.0567	2.33E-113	0.00E+00	0.00E+00	0.00E+00	0.00E+00
RL122V	24	9.8201	1.2951	5.65E-07	0.00E+00	0.00E+00	0.00E+00	0.00E+00
RG152Z	18	11.0103	6.2827	3.1183	1.1080	0.1176	1.73E-09	0.00E+00
RH152Z	60	36.7975	20.8155	9.9821	3.1537	0.1747	1.42E-21	0.00E+00
RH122Z	36	24.4358	16.3040	10.5594	6.4987	3.6477	1.6956	0.4800
RG130A	36	26.2572	18.0575	11.2255	5.7192	1.7526	0.0513	3.15E-52
RH152A	48	37.9341	29.2875	21.8520	15.4774	10.0675	5.5942	2.1670
RH122A	108	89.8792	74.4611	61.2704	49.9449	40.2040	31.8216	24.6133
RM120M	108	100.1458	92.8339	85.9916	79.5659	73.5192	67.8217	62.4494
RM-60M	324	241.1676	178.7885	131.5306	95.5914	68.2098	47.3440	31.4771
RASA15	18	11.4935	6.0324	1.9334	0.0684	9.72E-42	0.00E+00	0.00E+00
RBTR80	52	30.2256	15.4777	5.9028	0.8357	6.99E-06	0.00E+00	0.00E+00
RBTR60	12	6.7111	3.1899	1.0016	0.0552	4.32E-22	0.00E+00	0.00E+00
RT90	24	13.9088	6.0392	1.0464	0.0001	0.00E+00	0.00E+00	0.00E+00
RT72SM	54	33.6185	17.0140	4.9680	0.0948	2.79E-89	0.00E+00	0.00E+00
RT72S	36	25.8302	17.3811	10.3847	4.7981	1.0342	0.0012	0.00E+00
RT72M1	84	55.2195	31.6534	13.2666	1.9552	8.08E-06	0.00E+00	0.00E+00
RT55R	36	20.9336	9.3663	1.8929	0.0013	0.00E+00	0.00E+00	0.00E+00
RT55	144	101.9136	67.1548	38.6024	16.2539	2.4384	1.45E-05	0.00E+00
RZ30-X	24	8.2286	0.4091	9.49E-27	0.00E+00	0.00E+00	0.00E+00	0.00E+00
RZ234Z	30	17.3600	7.8019	1.6668	0.0024	0.00E+00	0.00E+00	0.00E+00
RGLLD	30	17.1341	7.4965	1.4625	0.0008	0.00E+00	0.00E+00	0.00E+00

Figure 5.5 After seven days of combat in posture "N" (Blue Attack vs. Red Hasty Defense), Red platforms have suffered substantial attrition.

Surviving Platforms

Platform Name	Initial Number	Time (t + 1)	Time (t + 2)	Time (t + 3)	Time (t + 4)	Time (t + 5)	Time (t + 6)	Time (t + 7)
FASCAM	1	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000
UH58D2	14	14.0000	14.0000	14.0000	14.0000	14.0000	14.0000	14.0000
UHCAH1	24	22.6039	21.7137	21.2020	20.9371	20.8059	20.7306	20.6906
UHC58D	60	57.4446	55.7618	54.7920	54.3527	54.1717	54.0606	53.9998
UHCL64	48	46.6175	45.6873	45.1036	44.7620	44.5623	44.4356	44.3655
UHCH64	48	46.1419	44.8930	44.0904	43.5808	43.2594	43.0547	42.9410
UACL64	210	209.0333	208.3562	207.9126	207.6385	207.4592	207.3151	207.2019
UACH64	138.63	138.0631	137.5838	137.1822	136.8498	136.5777	136.3584	136.1875
UA4F16	16.37	16.2260	16.1309	16.0688	16.0249	15.9925	15.9686	15.9520
UA3F16	23.88	23.7018	23.5938	23.5299	23.4855	23.4531	23.4295	23.4132
UA2F16	19	18.9408	18.8977	18.8674	18.8464	18.8301	18.8165	18.8056
UA1F16	20.75	20.5844	20.4747	20.4052	20.3608	20.3310	20.3095	20.2949
UA4A10	17.62	17.4843	17.3816	17.3056	17.2504	17.2098	17.1804	17.1604
UA3A10	20.13	19.9304	19.7928	19.6998	19.6350	19.5901	19.5596	19.5394
UA2A10	17	16.8677	16.7673	16.6938	16.6414	16.6018	16.5702	16.5461
UA1A10	19.25	19.0061	18.8338	18.7148	18.6313	18.5722	18.5297	18.4995
UMLRS	54	52.6119	51.8072	51.3774	51.1316	50.9956	50.9209	50.8687
UH155Z	288	285.2878	283.5185	282.3888	281.6743	281.2395	280.9640	280.7569
UM120Z	108	104.5513	102.1070	100.3739	99.1454	98.2797	97.6572	97.1849
UH155A	160	158.9410	158.2630	157.8460	157.5845	157.4251	157.3256	157.2536
UH105A	48	47.0385	46.5002	46.2146	46.0352	45.9202	45.8438	45.7853
UM-81M	36	34.4171	33.2759	32.4526	31.8636	31.4486	31.1582	30.9520
UM-60M	36	34.7715	33.8972	33.2764	32.8366	32.5297	32.3163	32.1640
UHMMWV	790	701.6925	641.6829	602.2999	577.5246	561.4168	549.8238	541.0231
UHMERJ	48	40.8038	36.1087	33.3383	31.9481	31.2606	30.8412	30.5567
UHMVT2	48	39.3818	33.8671	30.7186	29.2528	28.6144	28.2576	28.0123
UHMVST	96	85.4974	78.5142	74.0718	71.3873	69.6791	68.4383	67.4839
UHMV50	110	98.7864	91.3412	86.7272	84.0936	82.4605	81.2481	80.2966
UHMV19	108	95.6664	87.0745	81.2231	77.3355	74.6575	72.6440	71.0693
UM1A2	200	181.2800	169.1413	162.1737	158.8600	157.2761	156.2909	155.5098

Figure 5.6 After seven days of combat in posture “N” (Blue Attack vs. Red Hasty Defense), during which Blue was the attacker, Blue forces suffered only limited attrition.

In another example, Figures 5.7 and 5.8 depict numbers surviving for some platforms for seven days of combat in the SWA “H” posture (Red Attack vs. Blue Hasty Defense). From the posture description, one would expect to see far fewer estimated Red kills than in posture “N,” as well as an increased number of Blue estimated kills. As expected, Red forces suffered less estimated attrition than witnessed in posture “N,” and Blue forces fighting from a hasty defensive position suffered greater estimated attrition. It is useful to note that, although Red forces are offensive in posture “H,” some high-priority targets are still attrited to zero over the course of seven days.

Surviving Platforms

Insert the number of 24-hour periods you want to model: 7 — then click here

Platform Name	Initial Number	Time (t + 1)	Time (t + 2)	Time (t + 3)	Time (t + 4)	Time (t + 5)	Time (t + 6)	Time (t + 7)
RAD37A	117	101.9905	88.7692	76.9099	66.1274	56.2234	47.0585	38.5366
RAD57A	108	101.3154	95.0479	89.0747	83.3246	77.7526	72.3292	67.0337
RADSA8	0	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000
RAS18T	6	5.7552	5.5221	5.3000	5.0880	4.8854	4.6917	4.5062
RASA15	54	43.9896	35.1643	27.2507	20.1047	13.6741	8.0099	3.3591
RASA18	264	215.8812	178.8993	149.7456	126.2927	107.0998	91.1560	77.7364
RASA7	264	216.3639	179.6589	150.6586	127.2836	108.1231	92.1861	78.7611
RBFRDF	12	9.8506	8.2156	6.9283	5.8884	5.0321	4.3164	3.7109
RBFRDT	6	6.0000	6.0000	6.0000	6.0000	6.0000	6.0000	6.0000
RBMP3M	294	240.3956	197.3862	162.2003	132.9223	108.1714	86.9326	68.4646
RBMP5M	546	382.0201	247.3812	137.9380	54.3755	6.3812	2.74E-07	0.00E+00
RBMP5T	84	60.1723	37.9967	18.5007	4.2806	0.0080	0.00E+00	0.00E+00
RBRDM2	102	75.1270	53.1474	34.9966	20.1177	8.5323	1.3564	0.00E+00
RBRDM3	0	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000
RBRDM5	192	123.5328	71.0326	32.0377	7.1605	0.0204	0.00E+00	0.00E+00
RBTR60	36	23.3911	14.7312	8.6859	4.4766	1.6806	0.2131	0.00E+00
RBTR80	156	107.4098	73.5173	49.2438	31.5595	18.5867	9.1968	2.9207
RBUNKR	4	3.9715	3.9483	3.9291	3.9130	3.8993	3.8871	3.8760
RDSCAM	1	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000
REDMFD	1	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000
RFATP	9534	8731.0548	8061.8031	7485.5033	6977.8900	6523.4496	6111.7437	5735.3333
RFATPT	712	706.8860	701.9551	697.1561	692.4596	687.8470	683.3051	678.8242
RFDCVH	414	357.5603	310.6128	270.3451	235.0702	203.7023	175.5072	149.9610
RFDCVT	274	265.0275	256.1273	247.2843	238.4879	229.7302	221.0054	212.3086
RFOTP	684	579.6993	497.7006	431.3712	376.5230	330.3530	290.9082	256.7877
RFOTPT	68	66.7577	65.5486	64.3698	63.2192	62.0948	60.9949	59.9180
RFYAWN	12	9.8011	8.1333	6.8249	5.7725	4.9098	4.1922	3.5883
RFYAWT	6	6.0000	6.0000	6.0000	6.0000	6.0000	6.0000	6.0000
RG130A	108	94.3622	82.3995	71.7085	62.0305	53.1905	45.0662	37.5696
RG130Z	0	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000

Red | Red Update | Blue | Blue Update | COSAGE output | Platforms | Initial | Indirect | Kbr | Krb | LiveTgts

Figure 5.7 In the SWA "H" (Red Attack vs. Blue Hasty Defense) posture, Red forces, which are now attacking forces, suffer fewer estimated kills than in posture "N" (Blue Attack vs. Red Hasty Defense).

Surviving Platforms

Platform Name	Initial Number	Time (t + 1)	Time (t + 2)	Time (t + 3)	Time (t + 4)	Time (t + 5)	Time (t + 6)	Time (t + 7)
WAMFLD	0	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000
UH5802	7	7.0000	7.0000	7.0000	7.0000	7.0000	7.0000	7.0000
UHCAH1	12	9.1866	7.0297	5.4056	4.2084	3.3336	2.7060	2.2705
UHC58D	30	27.8761	26.2285	24.9799	24.0655	23.4239	23.0064	22.7695
UHCL64	24	20.6109	17.9346	15.8449	14.2415	13.0353	12.1577	11.5490
UHCH64	24	19.5757	16.1373	13.5182	11.5850	10.2214	9.3318	8.8239
UACL64	210.56	209.5695	208.7705	208.1375	207.6459	207.2671	206.9830	206.7840
UACH64	142	141.2539	140.6338	140.1178	139.6900	139.3384	139.0533	138.8264
UA4F16	15.5	15.3809	15.2876	15.2150	15.1585	15.1120	15.0725	15.0391
UA3F16	24.38	24.1696	24.0091	23.8893	23.8011	23.7333	23.6793	23.6371
UA2F16	19.62	19.5411	19.4757	19.4223	19.3798	19.3469	19.3227	19.3058
UA1F16	21.5	21.3215	21.1733	21.0505	20.9497	20.8680	20.8030	20.7522
UA4A10	18.5	18.3607	18.2441	18.1464	18.0648	17.9971	17.9415	17.8961
UA3A10	19.62	19.3758	19.1748	19.0100	18.8748	18.7620	18.6673	18.5882
UA2A10	16.37	16.1376	15.9385	15.7681	15.6227	15.4997	15.3964	15.3105
UA1A10	18.5	18.4703	18.4523	18.4438	18.4417	18.4417	18.4417	18.4417
UMLRS	27	19.4190	15.2191	12.7350	11.1918	10.1920	9.5311	9.0640
UH155Z	144	134.9833	128.1222	122.7814	118.5293	115.0606	112.1790	109.7118
UM120Z	54	43.7715	36.9441	32.1379	28.6077	25.9214	23.8242	22.1471
UH155A	80	74.3460	69.9534	66.4463	63.5821	61.1952	59.1728	57.4236
UH105A	24	21.9950	20.4957	19.3504	18.4564	17.7413	17.1573	16.6601
UM-81M	18	14.6922	12.3590	10.6434	9.3382	8.3168	7.5004	6.8353
UM-60M	18	14.5067	12.1110	10.3891	9.1007	8.1025	7.3115	6.6724
UHMMWV	395	285.2703	212.7553	162.4897	126.3813	99.7018	79.4889	63.7441
UHMERJ	24	17.6225	13.4274	10.5688	8.5788	7.1776	6.1830	5.4547
UHMVT2	24	17.5666	13.3102	10.3989	8.3671	6.9335	5.9127	5.1649
UHMVST	48	33.8499	24.7908	18.6811	14.3990	11.3084	9.0253	7.2975
UHMV50	55	41.5378	32.4207	25.9884	21.3194	17.8590	15.2452	13.2146
UHMV19	54	36.8267	26.2471	19.3218	14.5708	11.1884	8.7104	6.8471
UM1A2	100	84.3508	72.2288	62.7958	55.4590	49.7737	45.3761	41.9487

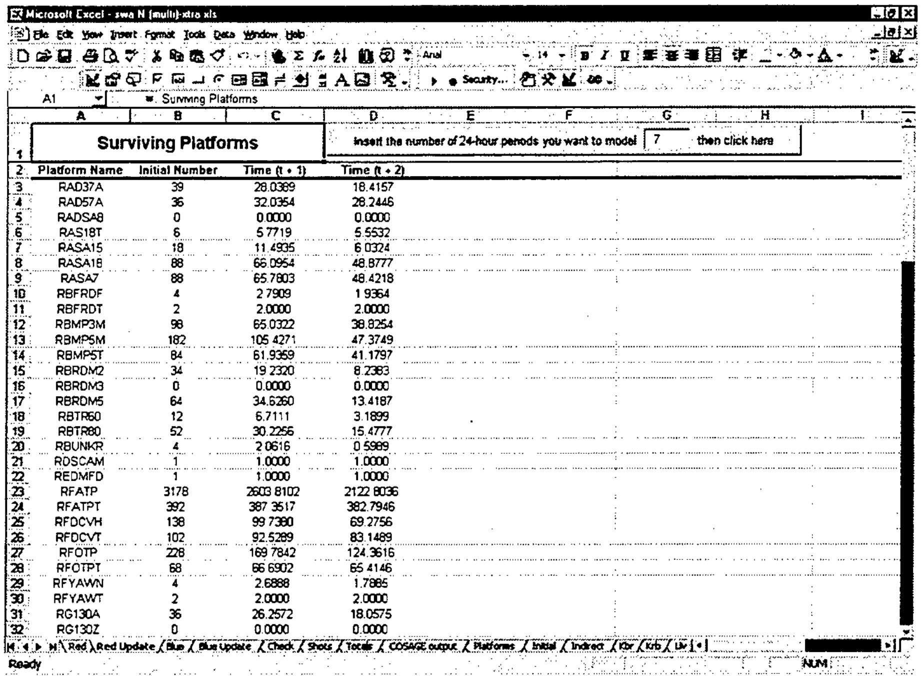
Red | Red Update | Blue | Blue Update | COSAGE output | Platforms | Initial | Indirect | Kbr | Krb | LiveTgts

Ready

Figure 5.8 In posture "H" (Red Attack vs. Blue Hasty Defense), Blue platforms, fighting from a defensive position, suffer significantly more estimated kills than in posture "N" (Blue Attack vs. Red Hasty Defense).

In postures “N” and “H,” various Red platforms are attrited to zero during seven days of combat, which may alarm the user. In the single-period implementation, the estimated number of targets killed for the first 48 hours of combat is compared to COSAGE output to ensure single-model plausibility. Once validated, the multiple-period implementation extrapolates the estimated kill rates for 24-hour periods beyond the initial 48 hours modeled in COSAGE. As a result, if the combat posture remains unchanged, estimated attrition rates calculated in the initial 48 hours are carried through successive 24-hour periods, producing seemingly high estimated kill rates.

In a final example, Figures 5.9 and 5.10 depict numbers surviving for some Red platforms for seven days of combat that starts in posture “N” (Blue Attack vs. Red Hasty Defense), then transitions to posture “F” (Blue Attack vs. Red Prepared Defense) after two days for the remainder of the seven days. As expected, attrition to some Red platforms slows when the transition is made to posture “F,” producing fewer estimated kills than seven days of combat in posture “N” (Blue Attack vs. Red Hasty Defense). However, because the analytical model implementation implicitly models the combat interactions within COSAGE, fire allocation associated with posture “F” produces more estimated kills for some Red platforms than in posture “N.”

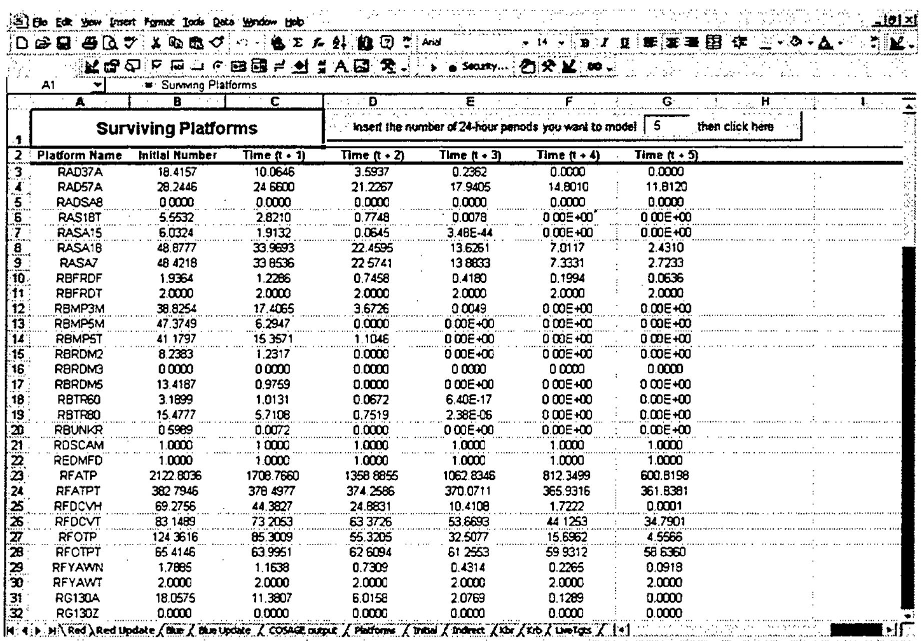


Surviving Platforms

Insert the number of 24-hour periods you want to model 7 then click here

Platform Name	Initial Number	Time (t + 1)	Time (t + 2)
RAD37A	39	28.0389	18.4157
RAD57A	36	32.0354	28.2446
RADSA8	0	0.0000	0.0000
RAS18T	6	5.7719	5.5532
RASA15	18	11.4935	6.0324
RASA18	88	66.0954	48.8777
RASA7	88	65.7803	48.4218
RBFRDF	4	2.7909	1.9364
RBFRDT	2	2.0000	2.0000
RBMP3M	98	65.0322	38.8254
RBMP5M	182	105.4271	47.3749
RBMP5T	84	61.9369	41.1797
RBRDM2	34	19.2320	8.2383
RBRDM3	0	0.0000	0.0000
RBRDM5	64	34.6260	13.4187
RBTR60	12	6.7111	3.1899
RBTR80	52	30.2256	15.4777
RBUNKR	4	2.0616	0.5989
RDSCAM	1	1.0000	1.0000
REDMFD	1	1.0000	1.0000
RFATP	3178	2603.8102	2122.8036
RFATPT	392	387.3517	382.7946
RFDCVH	138	99.7380	69.2756
RFDCVT	102	92.5289	83.1489
RFOTP	228	169.7842	124.3616
RFOTPT	68	66.6902	65.4146
RFYAWN	4	2.6888	1.7885
RFYAWT	2	2.0000	2.0000
RG130A	36	26.2572	18.0575
RG130Z	0	0.0000	0.0000

Figure 5.9 Combat begins in combat posture "N" (Blue Attack vs. Red Hasty Defense), then transitions to posture "F" (Blue Attack vs. Red Prepared Defense) after two days (see Figure 5.10 below).



Surviving Platforms

Insert the number of 24-hour periods you want to model 5 then click here

Platform Name	Initial Number	Time (t + 1)	Time (t + 2)	Time (t + 3)	Time (t + 4)	Time (t + 5)
RAD37A	18.4157	10.0646	3.5937	0.2362	0.0000	0.0000
RAD57A	28.2446	24.6600	21.2267	17.9405	14.8010	11.8120
RADSA8	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000
RAS18T	5.5532	2.8210	0.7748	0.0078	0.00E+00	0.00E+00
RASA15	6.0324	1.9132	0.0645	3.48E-44	0.00E+00	0.00E+00
RASA18	48.8777	33.9693	22.4595	13.6261	7.0117	2.4310
RASA7	48.4218	33.8536	22.5741	13.8833	7.3331	2.7233
RBFRDF	1.9364	1.2286	0.7458	0.4180	0.1994	0.0636
RBFRDT	2.0000	2.0000	2.0000	2.0000	2.0000	2.0000
RBMP3M	38.8254	17.4065	3.6726	0.0049	0.00E+00	0.00E+00
RBMP5M	47.3749	6.2947	0.0000	0.00E+00	0.00E+00	0.00E+00
RBMP5T	41.1797	15.3571	1.1046	0.00E+00	0.00E+00	0.00E+00
RBRDM2	8.2383	1.2317	0.0000	0.00E+00	0.00E+00	0.00E+00
RBRDM3	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000
RBRDM5	13.4187	0.9759	0.0000	0.00E+00	0.00E+00	0.00E+00
RBTR60	3.1899	1.0131	0.0672	6.40E-17	0.00E+00	0.00E+00
RBTR80	15.4777	5.7108	0.7519	2.38E-06	0.00E+00	0.00E+00
RBUNKR	0.5989	0.0072	0.0000	0.00E+00	0.00E+00	0.00E+00
RDSCAM	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000
REDMFD	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000
RFATP	2122.8036	1708.7660	1358.8855	1062.8346	812.3499	600.8198
RFATPT	382.7946	378.4977	374.2586	370.0711	365.9316	361.8381
RFDCVH	69.2756	44.3827	24.8831	10.4108	1.7222	0.0001
RFDCVT	83.1489	73.2053	63.3726	53.6693	44.1253	34.7901
RFOTP	124.3616	85.3009	55.3205	32.5077	15.6962	4.5566
RFOTPT	65.4146	63.9951	62.6094	61.2553	59.9312	58.6360
RFYAWN	1.7885	1.1638	0.7309	0.4314	0.2265	0.0918
RFYAWT	2.0000	2.0000	2.0000	2.0000	2.0000	2.0000
RG130A	18.0575	11.3807	6.0158	2.0769	0.1289	0.0000
RG130Z	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000

Figure 5.10 After two days of combat in posture "N" (Blue Attack vs. Red Hasty Defense), the battle transitions to posture "F" (Blue Attack vs. Red Prepared Defense) for the remaining five days of battle. Note that the Red platforms surviving at time (t + 1) in posture "N" become initial numbers in posture "F."

C. BENEFITS OF THE MODEL

1. Speed

Both the single-period and multiple-period models significantly reduce the time required to obtain results for 48 hours of ground combat. Once data is received from CAA in the form of COSAGE Killer-Victim Scoreboards, the primary time requirement involved when utilizing DTAM is converting files from Unix to Windows files. After the data is properly formatted, computer processing time involves a matter of minutes vice the hours required to run COSAGE. More importantly, the analytical model "fitted to" COSAGE data from the initial 48 hours allows suggestive extrapolation to much greater times without the necessity for additional COSAGE runs.

2. Sensitivity Analysis

Because DTAM runs relatively quickly, it can be used to explore numerous "what if" questions concerning parameters, including fire allocation, firing rates and initial platform numbers. Additionally, the relative speed at which DTAM produces results allows for code modifications to add functionality or to improve existing functions without investing vast amounts of time.

D. SUGGESTED FUTURE WORK

The implementation of the model presented in this thesis is very much a work in progress. The following items are candidates for such improvement.

1. Use Additional Data From COSAGE Runs

The model implementations presented in this thesis are based on Killer-Victim Scoreboard information (e.g., average number of shots and kills) that is the average of either sixteen or twenty COSAGE runs, depending on Blue-Red force interaction

frequency. Improved estimates of platform survivability could be obtained by using COSAGE data from individual runs. This could allow a better assessment of outcome *ranges* for much longer times without the need to run COSAGE.

2. Updating Current Data

The most immediate improvement to the model is achievable through updating the data sets. The data used for this thesis was drawn from Total Army Analysis 2005 (TAA05), which was superceded by TAA07 in late 1998. TAA07 data is currently available through the Center for Army Analysis (CAA), and will remain current into the year 2001, when TAA09 will be completed. The user may wish to update the parameters using the TAA07 data, or to make any other changes that may improve the functionality of the model.

3. Introduce Mine-Clearing Capabilities

DTAM does not currently model mine-clearing operations of any kind. As stated earlier, mines are currently modeled as one platform that can not be attrited firing indirect munitions. The inclusion of such operations may greatly enhance the models' usefulness when faced with artillery-fired mines, minefields, or both.

4. Make Shot Rates Adaptive to Changing Circumstances

As mentioned, once DTAM calculates shot rates for each shooter / munition / target present in the prescribed combat posture, it uses the same shot rates per shooter / munition / target every time the given posture is revisited. Because the COSAGE Killer-Victim Scoreboard does not describe fire re-allocation, modeling fire re-allocation will require an understanding of the inner workings of COSAGE code.

5. Resolve "Excess" Kills

As noted in Chapter II, when DTAM identifies a case in which the average number of kills recorded for a particular target exceeds the average total shots fired at that target for a given munition, kills are set equal to shots fired. Future work should seek a replacement for this simplifying assumption that does not discount these "excess" kills.

6. Improve the User Interface

Although neither the single-period nor the multiple-period models require excessive user intervention, improvement is always possible. It may be possible to restructure the Visual Basic code to improve its functionality, and to make its operation virtually transparent to the user.

E. ACHIEVEMENT OF OBJECTIVE

1. Reducing Modeling Time Requirements

DTAM computes estimated numbers of platforms surviving for the 48-hour period using COSAGE data. In addition, the model analytically projects number of platforms surviving far beyond the initial 48-hour period without the need for costly postprocessor simulations. While there are obvious possible limitations to the trustworthiness of DTAM, it greatly reduces computation time requirements, which makes it ideal for multiple runs and sensitivity analysis on an exploratory, if tentative, basis.

2. Producing Expected Numbers of Platforms Surviving

As shown in the results of this chapter, DTAM is capable of quickly providing reasonable estimates for expected numbers of platforms surviving (Red and Blue). It can

also be used to project munitions needed to sustain combat. Although the implementation does not represent every munition option available in today's Army, it does represent most present-day major weapon systems and their armament, and is easily updated as new systems become available and are modeled in COSAGE.

F. SUMMARY OF RESULTS

We have developed a tool for analysts to use in modeling ground combat that will produce expected numbers of platforms surviving using COSAGE output as initial input. This model allows analysts to conduct quick "what if" investigations by altering input parameters (e.g., shooting rates, initial platform numbers, etc.) between model runs. Although the model represents additional functionality to that already available through COSAGE and other simulations, there are still numerous improvements to be made. Some of these improvements have been outlined, and others will become apparent with experience.

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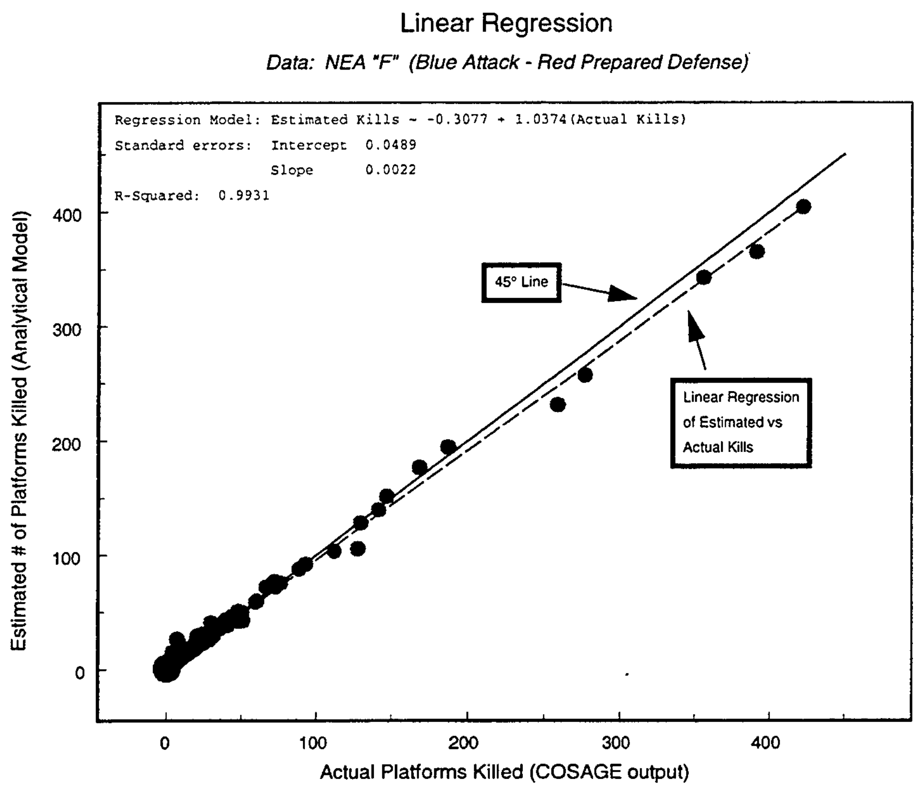
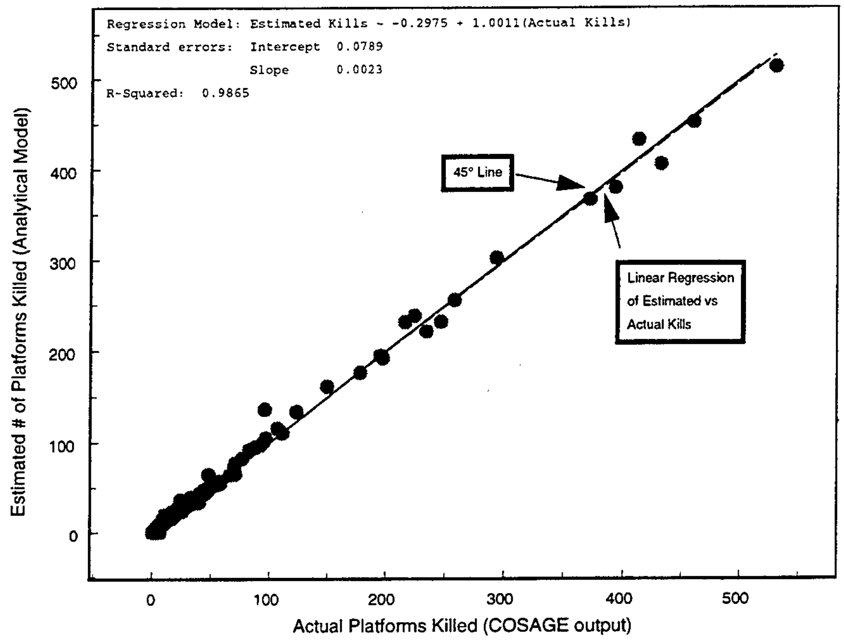
APPENDIX A. LINEAR REGRESSION MODELS

This appendix provides the reader one tool used to evaluate the plausibility of the single-period analytical model implementation. The linear regressions displayed compare estimated platform kills produced by the analytical model implementation during the initial 48 hours of battle to those kills produced by COSAGE. The linear regression model for combat posture NEA "D" (Blue Attack vs. Red Delay) presented in Chapter I is not displayed.



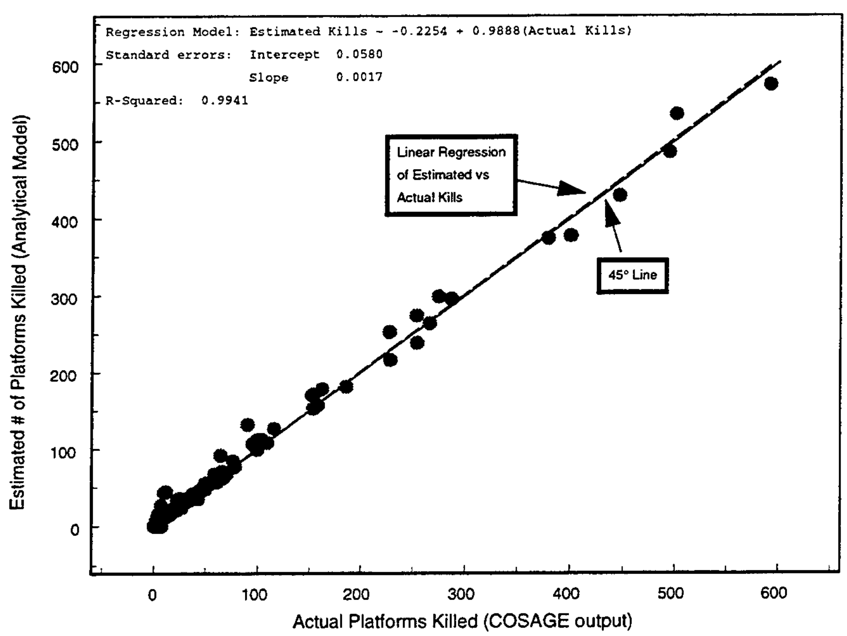
Linear Regression

Data: NEA "H" (Red Attack - Blue Hasty Defense)



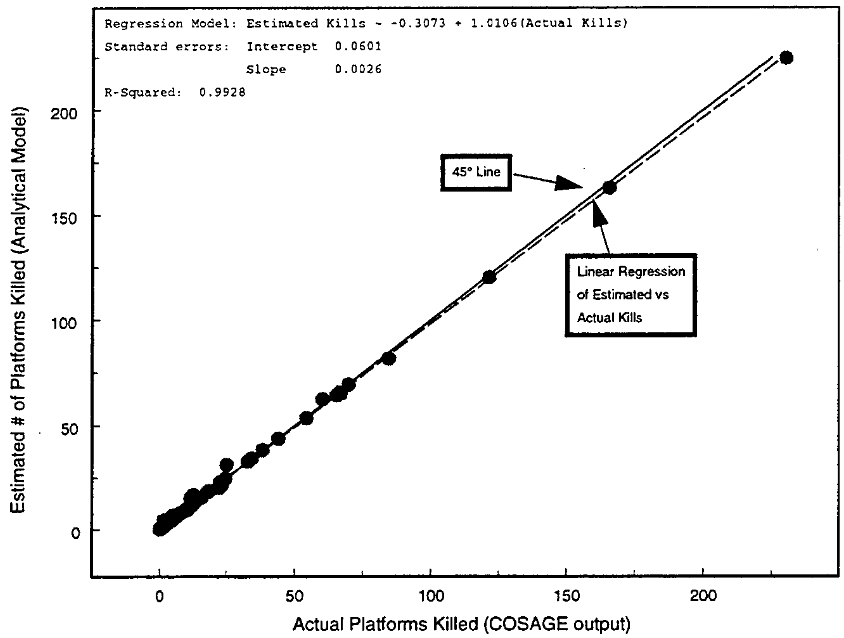
Linear Regression

Data: NEA "I" (Red Attack - Blue Prepared Defense)



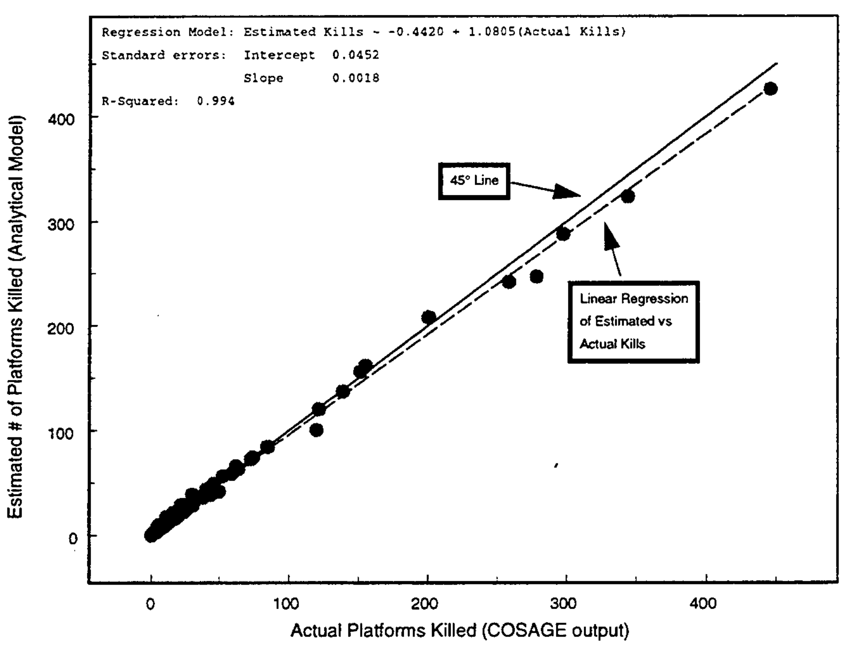
Linear Regression

Data: NEA "L" (Defense Light - Less Intense Static)



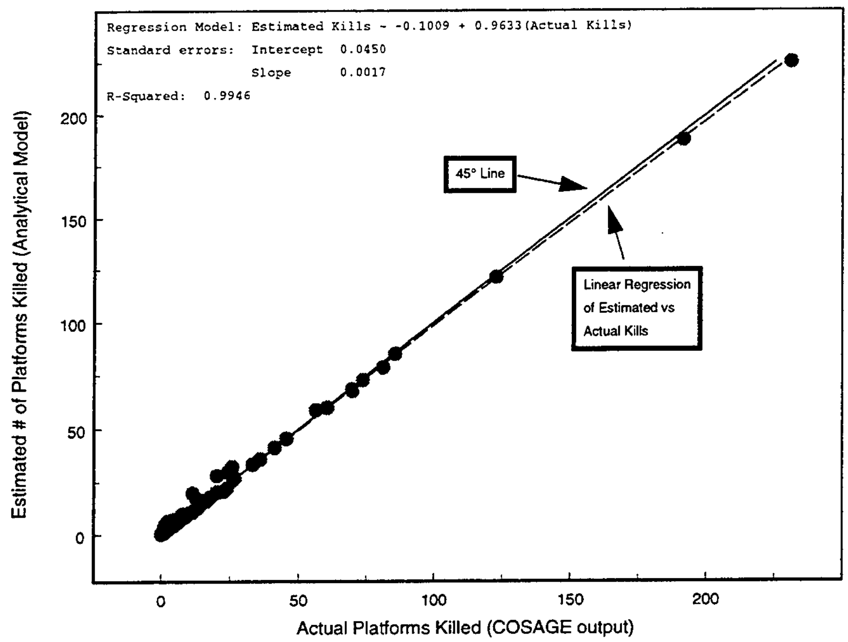
Linear Regression

Data: NEA "N" (Blue Attack - Red Hasty Defense)



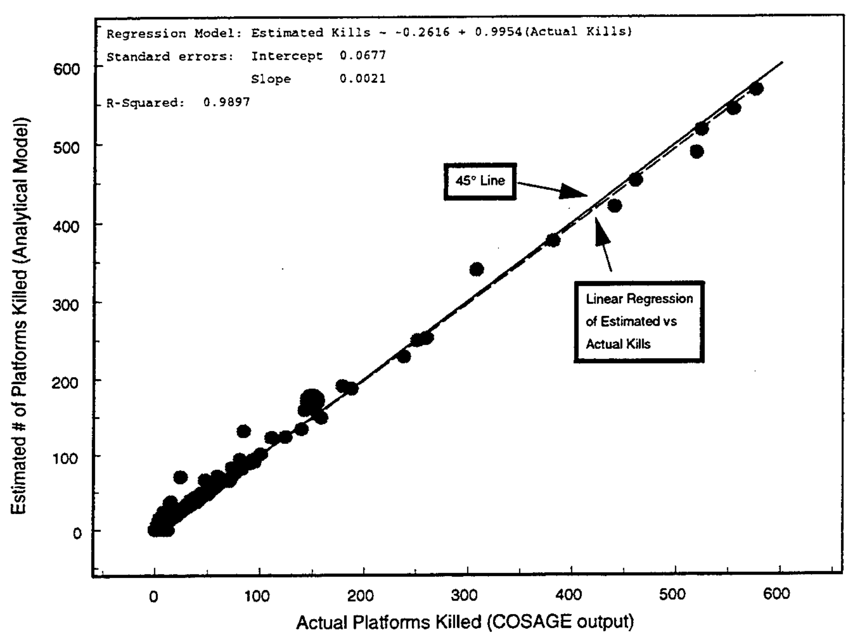
Linear Regression

Data: NEA "P" (Prep for Attack - Heavy Static)



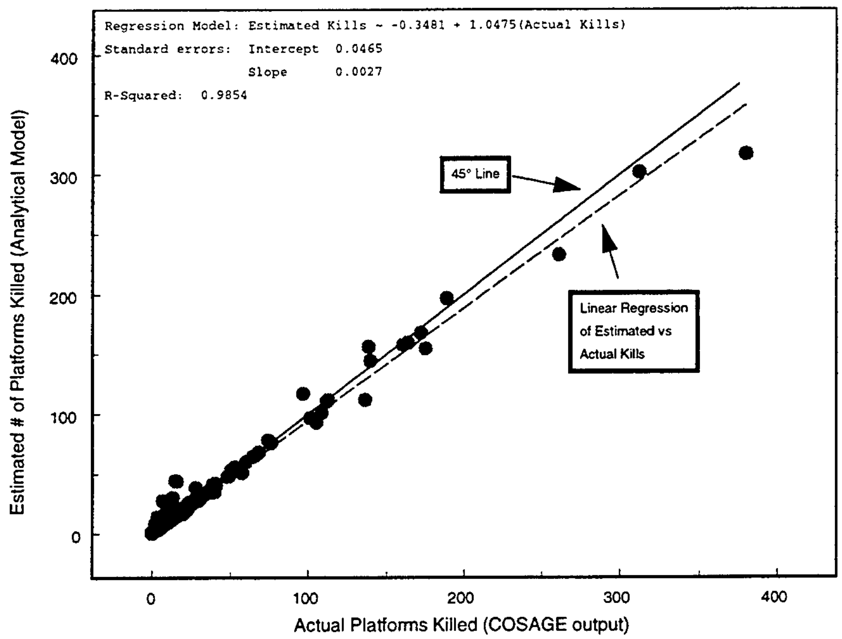
Linear Regression

Data: SWA "D" (Red Attack - Blue Delay)



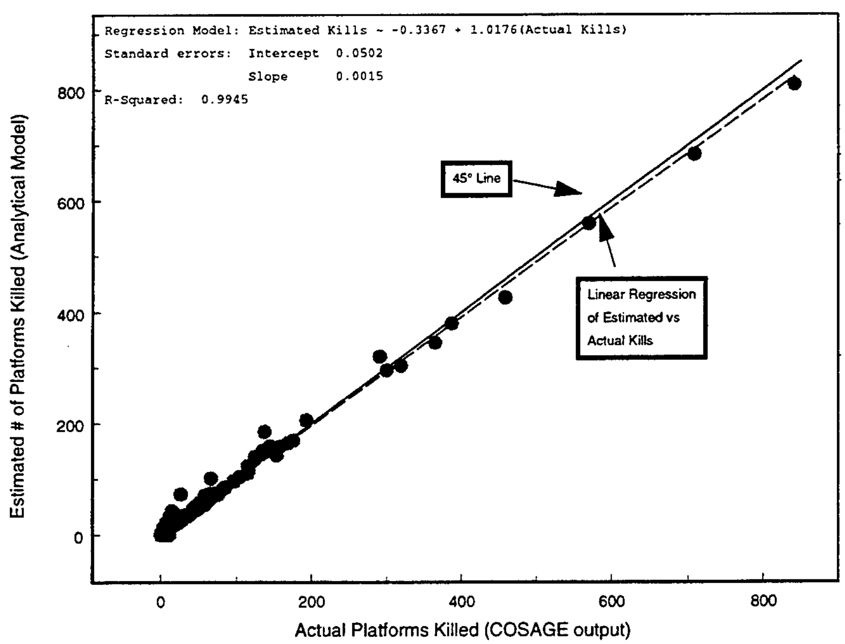
Linear Regression

Data: SWA "F" (Blue Attack - Red Prepared Defense)



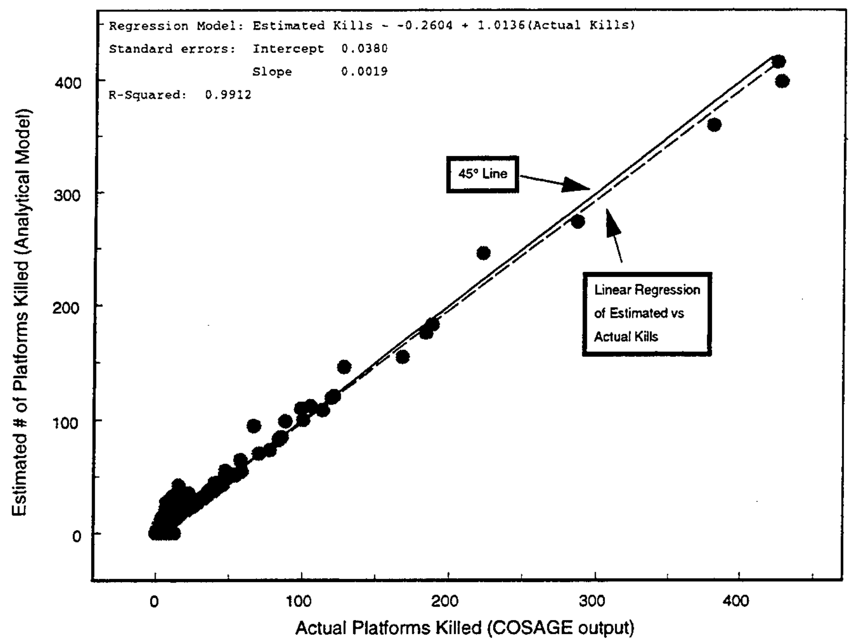
Linear Regression

Data: SWA "H" (Red Attack - Blue Hasty Defense)



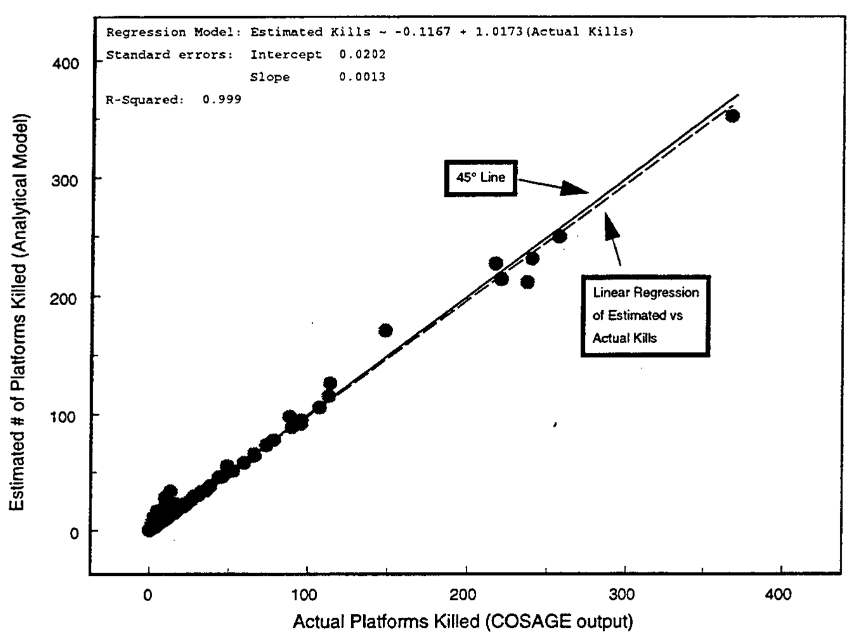
Linear Regression

Data: SWA "I" (Red Attack - Blue Prepared Defense)



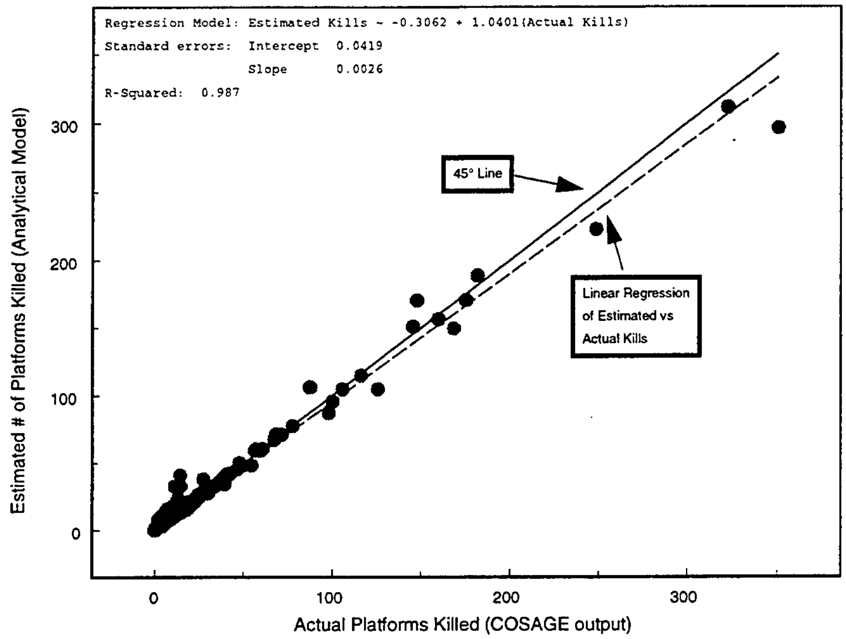
Linear Regression

Data: SWA "L" (Defense Light - Less Intense Static)



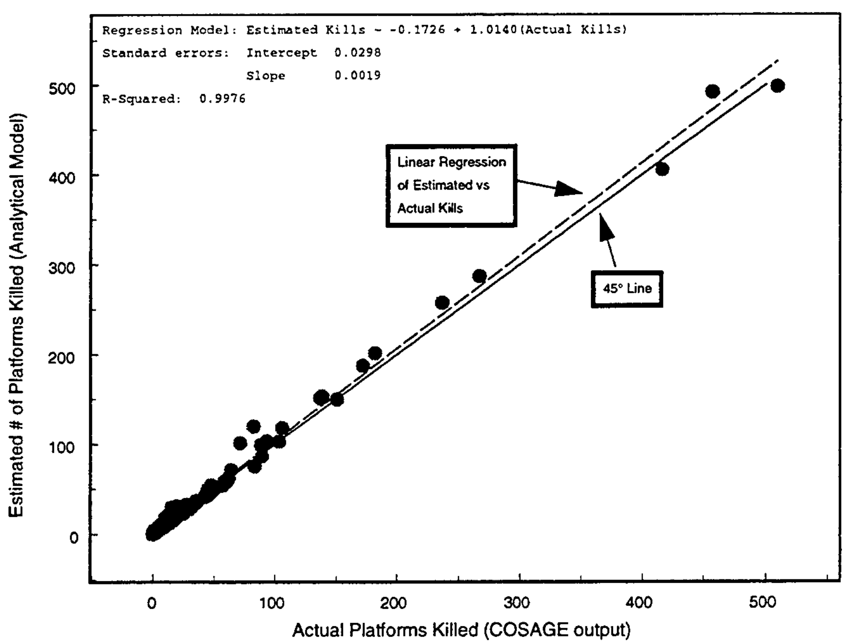
Linear Regression

Data: SWA "N" (Blue Attack - Red Hasty Defense)



Linear Regression

Data: SWA "P" (Prep for Attack - Heavy Static)



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APPENDIX B. SINGLE-PERIOD MODEL IMPLEMENTATION

The following is the Visual Basic single-period implementation of the analytical model formulation introduced by Professors Gaver and Jacobs.

```
Private Sub CommandButton1_Click()
'Parameters
Cosage = 1402
CosageRed = 552
RedUnits = 472
CosageBlue = 553
BlueUnits = 1319
InitialTotal = 204
InitialRed = 101
InitialBlue = 102
Reps = 20

'Sub A_initial()
'lists initial numbers of platforms on worksheet "Initial"
Worksheets("Initial").Range("a3:z50000") = "" 'ClearContents
rrow1 = 2
For i = 15 To Cosage
    Record = Worksheets("COSAGE output").Cells(i, 1)
    NextRecord = Worksheets("COSAGE output").Cells(i + 1, 1)
    If Record = "I" Then
        If NextRecord = "T" Then
            rrow1 = rrow1 + 1
            Worksheets("Initial").Cells(rrow1, 1) = Worksheets("COSAGE output").Cells(i, 4)
            Worksheets("Initial").Cells(rrow1, 2) = Worksheets("COSAGE output").Cells(i + 1, 3)
        Else
            rrow1 = rrow1 + 1
            Worksheets("Initial").Cells(rrow1, 1) = Worksheets("COSAGE output").Cells(i, 4)
            Worksheets("Initial").Cells(rrow1, 2) = Worksheets("COSAGE output").Cells(i, 2)
        End If
    End If
Next i
'End Sub

'Sub B_platforms()
'inserts the initial # of targets in column L and intial # of shooters in column M of "COSAGE output"
Worksheets("COSAGE output").Range("L15:Z5000") = "" 'ClearContents
For i = 15 To Cosage
    Record = Worksheets("COSAGE output").Cells(i, 1)
    Target = Worksheets("COSAGE output").Cells(i, 7)
    For j = 3 To InitialTotal
        Platform = Worksheets("Initial").Cells(j, 1)
        If Target = Platform Then
            Worksheets("COSAGE output").Cells(i, 12) = Worksheets("Initial").Cells(j, 2)
        End If
    Next j
Next i
'inserts initial # of shooters in column M of "COSAGE output"
For i = 15 To Cosage
```

```
    k = i
    value1 = Worksheets("COSAGE output").Cells(i, 1)
    Value2 = Worksheets("COSAGE output").Cells(i + 1, 1)
    If value1 = "I" Then
        If Value2 = "T" Then
            Init = Worksheets("COSAGE output").Cells(i + 1, 3)
        Else:
            Init = Worksheets("COSAGE output").Cells(i, 2)
        End If
    End If
    Do While (Worksheets("COSAGE output").Cells(k, 1) <> "I") And (Worksheets("COSAGE output").Cells(k, 1) <> Empty)
        Worksheets("COSAGE output").Cells(k, 13) = Init
        k = k + 1
    Loop
Next i
'End Sub

'Sub C_indirect()
'inserts total indirect shots in column N of "COSAGE output"
sumf = suma = summ = sumd = 0
For i = 15 To Cosage
    Record = Worksheets("COSAGE output").Cells(i, 1)
    Shooter = Worksheets("COSAGE output").Cells(i, 4)
    Weapon = Worksheets("COSAGE output").Cells(i, 5)
    Shots = Worksheets("COSAGE output").Cells(i, 8)
    If Record = "S" Then
        TotalShots = Shots
        If Weapon = "FASCAM" Then
            sumf = sumf + Shots
        End If
    ElseIf Record = "K" Then
        Worksheets("COSAGE output").Cells(i, 14) = TotalShots
    ElseIf Record = "D" Then
        If Shooter = "AIRMFD" Then
            suma = suma + Shots
        End If
        If Shooter = "FASCAM" Then
            Worksheets("COSAGE output").Cells(i, 14) = sumf
        End If
        If Shooter = "REDMFD" Then
            summ = summ + Shots
        End If
        If Shooter = "RDSCAM" Then
            sumd = sumd + Shots
        End If
    End If
Next i
```

```
For j = 15 To 2994
    newRecord = Worksheets("COSAGE output").Cells(j, 1)
    newShooter = Worksheets("COSAGE output").Cells(j, 4)
    If newRecord = "D" And newShooter = "AIRMFD" Then
        Worksheets("COSAGE output").Cells(j, 14) = suma
    End If
    If newRecord = "D" And newShooter = "REDMFD" Then
        Worksheets("COSAGE output").Cells(j, 14) = summ
    End If
    If newRecord = "D" And newShooter = "RDSCAM" Then
        Worksheets("COSAGE output").Cells(j, 14) = sumd
    End If
Next j
'End Sub

'Sub D_allocate()
'completes worksheet "indirect"
Worksheets("COSAGE output").Range("o15:z5000") = "" 'ClearContents
Worksheets("Indirect").Range("a3:z5000") = "" 'ClearContents
rrow2 = 2

For i = 15 To Cosage
    j = i
    If Worksheets("COSAGE output").Cells(i, 1) = "S" Then
        rrow2 = rrow2 + 1
        Worksheets("Indirect").Cells(rrow2, 1) = Worksheets("COSAGE output").Cells(i, 4) 'shooter
        Worksheets("Indirect").Cells(rrow2, 2) = Worksheets("COSAGE output").Cells(i, 5) 'weapon
        Worksheets("Indirect").Cells(rrow2, 3) = Worksheets("COSAGE output").Cells(i, 6) 'munition
        Sum = 0
        j = j + 1
        If Worksheets("COSAGE output").Cells(j, 1) <> "K" Then
            Do Until Worksheets("COSAGE output").Cells(j, 1) = "K"
                j = j + 1
            Loop
        End If
        Do Until Worksheets("COSAGE output").Cells(j, 1) <> "K"
            Sum = Sum + Worksheets("COSAGE output").Cells(j, 8)
            j = j + 1
        Loop
        Worksheets("Indirect").Cells(rrow2, 4) = Sum
    End If
Next i
'End Sub

'Sub e_Kbr()
'creates a listing of Kbr values needed for EKP calculations
Worksheets("Kbr").Range("a3:z20000") = "" 'ClearContents
rrow3 = 2
```

```
For i = 3 To InitialRed
myString1 = Worksheets("Initial").Cells(i, 1) 'pull Red platform names from "Initial"
    For j = CosageBlue To BlueUnits 'Blue shooters
        Record = Worksheets("COSAGE output").Cells(j, 1)
        Shooter = Worksheets("COSAGE output").Cells(j, 4)
        Weapon = Worksheets("COSAGE output").Cells(j, 5)
        Munition = Worksheets("COSAGE output").Cells(j, 6)
        Shots = Worksheets("COSAGE output").Cells(j, 8)
        Kills = Worksheets("COSAGE output").Cells(j, 9)
        DeadKills = Worksheets("COSAGE output").Cells(j, 10)
        Sbr = Worksheets("COSAGE output").Cells(j, 14)

        myString2 = Worksheets("COSAGE output").Cells(j, 7) 'pull Red target names from "COSAGE output"
        If myString1 = myString2 Then
            rrow3 = rrow3 + 1
            Worksheets("Kbr").Cells(rrow3, 1) = Worksheets("Initial").Cells(i, 1)    'target
            Worksheets("Kbr").Cells(rrow3, 2) = Worksheets("Initial").Cells(i, 2)    'target density
            Worksheets("Kbr").Cells(rrow3, 3) = Shooter                              'shooter
            Worksheets("Kbr").Cells(rrow3, 4) = Munition                             'munition
            If Munition <> "PGM" Then
                Worksheets("Kbr").Cells(rrow3, 6) = Sbr                              'Sbr
            End If
            Worksheets("Kbr").Cells(rrow3, 10) = Kills                               'kills
            Worksheets("Kbr").Cells(rrow3, 13) = Record                              'record
            If Record = "D" Or Shooter = "UMLRS" Or Munition = "PGM" Then
                Worksheets("Kbr").Cells(rrow3, 7) = Shots
                If DeadKills = "0" Or DeadKills = Empty Then
                    Worksheets("Kbr").Cells(rrow3, 11) = "0"
                Else: Worksheets("Kbr").Cells(rrow3, 11) = DeadKills
                End If
                If Worksheets("Kbr").Cells(rrow3, 7) <> "0" Then
                    Worksheets("Kbr").Cells(rrow3, 12) = (Kills - DeadKills) / Shots
                Else
                    Worksheets("Kbr").Cells(rrow3, 12) = "N/A"
                End If
            Else: Worksheets("Kbr").Cells(rrow3, 12) = "Indirect"
            End If
        End If
    Next j
Next i

newRow1 = 2 'inserts initial # of shooters in column E of "Kbr"
For k = 3 To rrow3
    BlueShooter = Worksheets("Kbr").Cells(k, 3)
    BlueMunition = Worksheets("Kbr").Cells(k, 4)
    BlueRecord = Worksheets("Kbr").Cells(k, 13)
    myString3 = BlueShooter
    For m = InitialBlue To InitialTotal
```

```
            myString4 = Worksheets("Initial").Cells(m, 1)
            If myString3 = myString4 Then
                newRow1 = newRow1 + 1
                If newRow1 <= rrow3 Then
                    Worksheets("Kbr").Cells(newRow1, 5) = Worksheets("Initial").Cells(m, 2)
                    If Worksheets("Kbr").Cells(newRow1, 5) <> "0" Then
                        If BlueRecord = "D" Or BlueShooter = "UMLRS" Or BlueMunition = "PGM" Then
                            Worksheets("Kbr").Cells(newRow1, 8) = Worksheets("Kbr").Cells(newRow1, 7) /
                                                                  Worksheets("Kbr").Cells(newRow1, 5)
                        ElseIf BlueRecord = "K" Then
                            Worksheets("Kbr").Cells(newRow1, 8) = Worksheets("Kbr").Cells(newRow1, 6) /
                                                                  Worksheets("Kbr").Cells(newRow1, 5)
                        End If
                        Worksheets("Kbr").Cells(newRow1, 9) = Worksheets("Kbr").Cells(newRow1, 8) / 2
                    End If
                End If
            End If
        Next m
    Next k

    For n = 3 To rrow3
        R0 = Worksheets("Kbr").Cells(n, 2)
        B0 = Worksheets("Kbr").Cells(n, 5)
        Sbr = Worksheets("Kbr").Cells(n, 6)
        NNbr0 = Worksheets("Kbr").Cells(n, 7)
        nbr0 = Worksheets("Kbr").Cells(n, 8)
        Kbr0 = Worksheets("Kbr").Cells(n, 10)
        KbrRecord = Worksheets("Kbr").Cells(n, 13)
        KbrShooter = Worksheets("Kbr").Cells(n, 3)
        KbrMunition = Worksheets("Kbr").Cells(n, 4)

        If (KbrRecord = "D" Or KbrShooter = "UMLRS" Or KbrMunition = "PGM") And R0 <> 0 And KbrMunition <> "MINES" Then
            If Kbr0 <= NNbr0 Then
                'Bayes direct (one shot)
                Worksheets("Kbr").Cells(n, 14) = (((B0 * nbr0) - Kbr0 + (1 / Reps)) / ((B0 * nbr0) + (2 / Reps)))
                Worksheets("Kbr").Cells(n, 15) = (Worksheets("Kbr").Cells(n, 14)) ^ (B0 * nbr0 / R0) '(all shots - 2 days)
                Worksheets("Kbr").Cells(n, 16) = Sqr(Worksheets("Kbr").Cells(n, 15)) '(all shots - one day)
            ElseIf Kbr0 >= NNbr0 Then
                Kbr0 = NNbr0
                'Bayes direct (one shot)
                Worksheets("Kbr").Cells(n, 14) = (((B0 * nbr0) - Kbr0 + (1 / Reps)) / ((B0 * nbr0) + (2 / Reps)))
                Worksheets("Kbr").Cells(n, 15) = (Worksheets("Kbr").Cells(n, 14)) ^ (B0 * nbr0 / R0) '(all shots - 2 days)
                Worksheets("Kbr").Cells(n, 16) = Sqr(Worksheets("Kbr").Cells(n, 15)) '(all shots - one day)
            End If
            If B0 <> 0 And B0 <> Empty Then
                'Expected kill rate (direct fire)
                Worksheets("Kbr").Cells(n, 17) = (((B0 * nbr0) / B0) * ((Kbr0 + (1 / Reps)) / (nbr0 + (2 / Reps)))) / 2
            End If
```

```
    ElseIf KbrRecord = "K" Or KbrMunition = "MINES" Then
        Worksheets("Kbr").Cells(n, 14) = (1 - ((Kbr0 + (1 / Reps)) / ((Sbr * R0) + (2 / Reps)))) 'Bayes indirect fire (one shot)
        Worksheets("Kbr").Cells(n, 15) = (Worksheets("Kbr").Cells(n, 14)) ^ (Sbr) '(all shots - 2 days)
        Worksheets("Kbr").Cells(n, 16) = Sqr(Worksheets("Kbr").Cells(n, 15)) '(all shots - one day)
        If B0 <> 0 Then
            'Expected kill rate (indirect fire)
            Worksheets("Kbr").Cells(n, 17) = ((Sbr / B0) * ((Kbr0 + (1 / Reps)) / ((Sbr * R0) + (2 / Reps)))) / 2
        End If
    End If
Next n
'End Sub

'Sub f_Krb()
'creates a listing of Krb values needed for EKP calculations
Worksheets("Krb").Range("a3:z20000") = "" 'ClearContents
rrow4 = 2
For i = InitialBlue To InitialTotal
myString1 = Worksheets("Initial").Cells(i, 1) 'pull Blue platform names from "Initial"
    For j = 15 To RedUnits 'Red shooters
        Record = Worksheets("COSAGE output").Cells(j, 1)
        Shooter = Worksheets("COSAGE output").Cells(j, 4)
        Weapon = Worksheets("COSAGE output").Cells(j, 5)
        Munition = Worksheets("COSAGE output").Cells(j, 6)
        Shots = Worksheets("COSAGE output").Cells(j, 8)
        Kills = Worksheets("COSAGE output").Cells(j, 9)
        DeadKills = Worksheets("COSAGE output").Cells(j, 10)
        Sbr = Worksheets("COSAGE output").Cells(j, 14)

        myString2 = Worksheets("COSAGE output").Cells(j, 7) 'pull Blue target names from "COSAGE output"
        If myString1 = myString2 Then
            rrow4 = rrow4 + 1
            Worksheets("Krb").Cells(rrow4, 1) = Worksheets("Initial").Cells(i, 1)    'target
            Worksheets("Krb").Cells(rrow4, 2) = Worksheets("Initial").Cells(i, 2)    'target density
            Worksheets("Krb").Cells(rrow4, 3) = Shooter                              'shooter
            Worksheets("Krb").Cells(rrow4, 4) = Munition                             'munition
            If Shooter <> "UMLRS" And Munition <> "PGM" Then
                Worksheets("Krb").Cells(rrow4, 6) = Sbr                              'Sbr
            End If
            Worksheets("Krb").Cells(rrow4, 10) = Kills                               'kills
            Worksheets("Krb").Cells(rrow4, 13) = Record                              'record
            If Record = "D" Or Shooter = "UMLRS" Or Munition = "PGM" Then
                Worksheets("Krb").Cells(rrow4, 7) = Shots
                If DeadKills = "0" Or DeadKills = Empty Then
                    Worksheets("Krb").Cells(rrow4, 11) = "0"
                Else: Worksheets("Krb").Cells(rrow4, 11) = DeadKills
                End If
                If Worksheets("Krb").Cells(rrow4, 7) <> "0" Then
                    Worksheets("Krb").Cells(rrow4, 12) = (Kills - DeadKills) / Shots
```

```
                Else
                    Worksheets("Krb").Cells(rrow4, 12) = "*****"
                End If
            Else: Worksheets("Krb").Cells(rrow4, 12) = "Indirect"
            End If
        End If
    Next j
Next i

newRow2 = 2 'inserts initial # of shooters in column E of "Krb"
For k = 3 To rrow4
    RedShooter = Worksheets("Krb").Cells(k, 3)
    RedMunition = Worksheets("Krb").Cells(k, 4)
    RedRecord = Worksheets("Krb").Cells(k, 13)
    myString3 = RedShooter
    For m = 3 To InitialRed
        myString4 = Worksheets("Initial").Cells(m, 1)
        If myString3 = myString4 Then
            newRow2 = newRow2 + 1
            If newRow2 <= rrow4 Then
                Worksheets("Krb").Cells(newRow2, 5) = Worksheets("Initial").Cells(m, 2)
                If Worksheets("Krb").Cells(newRow2, 5) <> "0" Then
                    If RedRecord = "D" Or RedShooter = "UMLRS" Or RedMunition = "PGM" Then
                        Worksheets("Krb").Cells(newRow2, 8) = Worksheets("Krb").Cells(newRow2, 7) /
                                                              Worksheets("Krb").Cells(newRow2, 5)
                    ElseIf RedRecord = "K" Then
                        Worksheets("Krb").Cells(newRow2, 8) = Worksheets("Krb").Cells(newRow2, 6) /
                                                              Worksheets("Krb").Cells(newRow2, 5)
                    End If
                    Worksheets("Krb").Cells(newRow2, 9) = Worksheets("Krb").Cells(newRow2, 8) / 2
                End If
            End If
        End If
    Next m
Next k

For n = 3 To rrow4
    B0 = Worksheets("Krb").Cells(n, 2)
    R0 = Worksheets("Krb").Cells(n, 5)
    Srb = Worksheets("Krb").Cells(n, 6)
    NNrb0 = Worksheets("Krb").Cells(n, 7)
    nrb0 = Worksheets("Krb").Cells(n, 8)
    Krb0 = Worksheets("Krb").Cells(n, 10)
    KrbRecord = Worksheets("Krb").Cells(n, 13)
    KrbShooter = Worksheets("Krb").Cells(n, 3)
    KrbMunition = Worksheets("Krb").Cells(n, 4)

    If (KrbRecord = "D" Or KrbShooter = "UMLRS" Or KrbMunition = "PGM") And B0 <> 0 And KrbMunition <> "MINES" Then
```

```
        If Krb0 <= NNrb0 Then
            'Bayes direct (one shot)
            Worksheets("Krb").Cells(n, 14) = (((R0 * nrb0) - Krb0 + (1 / Reps)) / ((R0 * nrb0) + (2 / Reps)))
            Worksheets("Krb").Cells(n, 15) = (Worksheets("Krb").Cells(n, 14)) ^ (R0 * nrb0 / B0) '(all shots - 2 days)
            Worksheets("Krb").Cells(n, 16) = Sqr(Worksheets("Krb").Cells(n, 15)) '(all shots - one day)
        ElseIf Krb0 >= NNrb0 Then
            Krb0 = NNrb0
            'Bayes direct (one shot)
            Worksheets("Krb").Cells(n, 14) = (((R0 * nrb0) - Krb0 + (1 / Reps)) / ((R0 * nrb0) + (2 / Reps)))
            Worksheets("Krb").Cells(n, 15) = (Worksheets("Krb").Cells(n, 14)) ^ (R0 * nrb0 / B0) '(all shots - 2 days)
            Worksheets("Krb").Cells(n, 16) = Sqr(Worksheets("Krb").Cells(n, 15)) '(all shots - one day)
        End If
        If R0 <> 0 And R0 <> Empty Then
            'Expected kill rate (direct fire)
            Worksheets("Krb").Cells(n, 17) = (((R0 * nrb0) / R0) * ((Krb0 + (1 / Reps)) / (nrb0 + (2 / Reps)))) / 2
        End If
    ElseIf KrbRecord = "K" Or KrbMunition = "MINES" Then
        Worksheets("Krb").Cells(n, 14) = (1 - ((Krb0 + (1 / Reps)) / ((Srb * B0) + (2 / Reps)))) 'Bayes indirect fire (one shot)
        Worksheets("Krb").Cells(n, 15) = (Worksheets("Krb").Cells(n, 14)) ^ (Srb) '(all shots - 2 days)
        Worksheets("Krb").Cells(n, 16) = Sqr(Worksheets("Krb").Cells(n, 15)) '(all shots - one day)
        If R0 <> 0 Then
            'Expected kill rate (indirect fire)
            Worksheets("Krb").Cells(n, 17) = ((Srb / R0) * ((Krb0 + (1 / Reps)) / ((Srb * B0) + (2 / Reps)))) / 2
        End If
    End If
Next n
'End Sub

'Sub G_LiveTgts()
'populates "LiveTgts, comparing estimated platforms killed to observed kills
Worksheets("LiveTgts").Range("a3:z50000") = "" 'ClearContents
rrow5 = 3
For i = 2 To InitialRed
Plive = 1
Kills = 0
myString1 = Worksheets("Initial").Cells(i, 1) 'pull Red platform names from "Initial"
    For j = 2 To 2000
        Record = Worksheets("Kbr").Cells(j, 13)
        myString2 = Worksheets("Kbr").Cells(j, 1) 'target names
        If myString1 = myString2 Then
            Worksheets("LiveTgts").Cells(rrow5, 2) = myString1                         'target
            Worksheets("LiveTgts").Cells(rrow5, 1) = Record                            'record type
            Worksheets("LiveTgts").Cells(rrow5, 3) = Worksheets("Kbr").Cells(j, 2)     'initial #
            Worksheets("LiveTgts").Cells(rrow5, 4) = Worksheets("Kbr").Cells(j, 3)     'shooter
            If Record <> "K" Then
                Worksheets("LiveTgts").Cells(rrow5, 5) = Worksheets("Kbr").Cells(j, 7) 'Nbr(0)
            End If
            Worksheets("LiveTgts").Cells(rrow5, 6) = Worksheets("Kbr").Cells(j, 6)     'Sbr
```

```
            Worksheets("LiveTgts").Cells(rrow5, 7) = Worksheets("Kbr").Cells(j, 4)    'munition
            Worksheets("LiveTgts").Cells(rrow5, 8) = Worksheets("Kbr").Cells(j, 10)   'kills
            ObsKills = Worksheets("LiveTgts").Cells(rrow5, 8)
            If Record = "D" Or Record = "K" Then
                Worksheets("LiveTgts").Cells(rrow5, 9) = Worksheets("Kbr").Cells(j, 15) 'Bayes q(2) estimate
            End If
            Plive = Plive * Worksheets("LiveTgts").Cells(rrow5, 9) 'multiplicative P(survive) function
            Worksheets("LiveTgts").Cells(rrow5, 10) = 1 - Worksheets("LiveTgts").Cells(rrow5, 9) 'P(kill)
            'Estimated kills (48 hours)
            Worksheets("LiveTgts").Cells(rrow5, 11) = Worksheets("LiveTgts").Cells(rrow5, 10) *
                                                  Worksheets("LiveTgts").Cells(rrow5, 3)
            Worksheets("LiveTgts").Cells(rrow5, 12) = Worksheets("LiveTgts").Cells(rrow5, 11) / 2 'est kills (24 hours)
            Worksheets("LiveTgts").Cells(rrow5, 13) = Plive
            Worksheets("LiveTgts").Cells(rrow5, 14) = (Plive * Worksheets("LiveTgts").Cells(rrow5, 3)) 'est surviving platforms
            'Estrimated cumulative kills
            Worksheets("LiveTgts").Cells(rrow5, 15) = Worksheets("LiveTgts").Cells(rrow5, 3) -
                                                  Worksheets("LiveTgts").Cells(rrow5, 14)
            Kills = Kills + Worksheets("LiveTgts").Cells(rrow5, 8)
            Worksheets("LiveTgts").Cells(rrow5, 16) = Kills
            Worksheets("LiveTgts").Cells(rrow5, 20) = Worksheets("LiveTgts").Cells(rrow5, 15) - Kills 'Over-estimation
            Worksheets("LiveTgts").Cells(rrow5, 17) = Worksheets("LiveTgts").Cells(rrow5, 11) - ObsKills 'estimate error
            If Kills <> "0" Then
                Worksheets("LiveTgts").Cells(rrow5, 21) = ((Worksheets("LiveTgts").Cells(rrow5, 15) - Kills) / Kills)
            End If
            If ObsKills <> 0 Then
                Worksheets("LiveTgts").Cells(rrow5, 18) = (Worksheets("LiveTgts").Cells(rrow5, 11) - ObsKills) / ObsKills
            End If
            rrow5 = rrow5 + 1
        End If
    Next j
    myRow1 = Worksheets("LiveTgts").Cells(rrow5 - 1, 3)
    myRow2 = Worksheets("LiveTgts").Cells(rrow5, 3)
    If myRow1 <> myRow2 Then
        rrow5 = rrow5 + 1
    End If
Next i

rrow5 = rrow5 + 1
For i = InitialBlue To InitialTotal
Plive = 1
Kills = 0
myString1 = Worksheets("Initial").Cells(i, 1) 'pull Blue platform names from "Initial"
    For j = 2 To 2000
        Record = Worksheets("Krb").Cells(j, 13)
        myString2 = Worksheets("Krb").Cells(j, 1) 'target names
        If myString1 = myString2 Then
            Worksheets("LiveTgts").Cells(rrow5, 2) = myString1
            Worksheets("LiveTgts").Cells(rrow5, 1) = Record                          'Record type
```

```
            Worksheets("LiveTgts").Cells(rrow5, 3) = Worksheets("Krb").Cells(j, 2)    'Initial #
            Worksheets("LiveTgts").Cells(rrow5, 4) = Worksheets("Krb").Cells(j, 3)    'Shooter
            If Record <> "K" Then
                Worksheets("LiveTgts").Cells(rrow5, 5) = Worksheets("Krb").Cells(j, 7) 'Nbr(0)
            End If
            Worksheets("LiveTgts").Cells(rrow5, 6) = Worksheets("Krb").Cells(j, 6)    'Sbr
            Worksheets("LiveTgts").Cells(rrow5, 7) = Worksheets("Krb").Cells(j, 4)    'Munition
            Worksheets("LiveTgts").Cells(rrow5, 8) = Worksheets("Krb").Cells(j, 10)   'Kills
            ObsKills = Worksheets("LiveTgts").Cells(rrow5, 8)
            If Record = "D" Or Record = "K" Then
                Worksheets("LiveTgts").Cells(rrow5, 9) = Worksheets("Krb").Cells(j, 15) 'Bayes q(2) estimate
            End If
            Plive = Plive * Worksheets("LiveTgts").Cells(rrow5, 9) 'multiplicative P(survive) function
            Worksheets("LiveTgts").Cells(rrow5, 10) = 1 - Worksheets("LiveTgts").Cells(rrow5, 9) 'P(kill)
            'Estimated kills (48 hours)
            Worksheets("LiveTgts").Cells(rrow5, 11) = Worksheets("LiveTgts").Cells(rrow5, 10) *
                                                      Worksheets("LiveTgts").Cells(rrow5, 3)
            Worksheets("LiveTgts").Cells(rrow5, 12) = Worksheets("LiveTgts").Cells(rrow5, 11) / 2 'est kills (24 hours)
            Worksheets("LiveTgts").Cells(rrow5, 13) = Plive
            Worksheets("LiveTgts").Cells(rrow5, 14) = (Plive * Worksheets("LiveTgts").Cells(rrow5, 3)) 'est surviving platforms
            'Estimated cumulative kills
            Worksheets("LiveTgts").Cells(rrow5, 15) = Worksheets("LiveTgts").Cells(rrow5, 3) -
                                                      Worksheets("LiveTgts").Cells(rrow5, 14)
            Kills = Kills + Worksheets("LiveTgts").Cells(rrow5, 8)
            Worksheets("LiveTgts").Cells(rrow5, 16) = Kills
            Worksheets("LiveTgts").Cells(rrow5, 20) = Worksheets("LiveTgts").Cells(rrow5, 15) - Kills 'Over-estimation
            Worksheets("LiveTgts").Cells(rrow5, 17) = Worksheets("LiveTgts").Cells(rrow5, 11) - ObsKills 'estimate error
            If Kills <> "0" Then
                Worksheets("LiveTgts").Cells(rrow5, 21) = ((Worksheets("LiveTgts").Cells(rrow5, 15) - Kills) / Kills)
            End If
            If ObsKills <> 0 Then
                Worksheets("LiveTgts").Cells(rrow5, 18) = (Worksheets("LiveTgts").Cells(rrow5, 11) - ObsKills) / ObsKills
            End If
            rrow5 = rrow5 + 1
        End If
    Next j
    myRow1 = Worksheets("LiveTgts").Cells(rrow5 - 1, 3)
    myRow2 = Worksheets("LiveTgts").Cells(rrow5, 3)
    If myRow1 <> myRow2 Then
        rrow5 = rrow5 + 1
    End If
Next i
'End Sub

'Sub H_expend()
Worksheets("Munitions").Range("a3:z50000") = "" 'ClearContents
Worksheets("COSAGE output").Range("o15:z50000") = "" 'ClearContents
rrow6 = 2
```

```
For i = 901 To 1023
    Sum = 0
    Munition = Worksheets("Platforms").Cells(i, 4)
    For j = 15 To Cosage
        Record = Worksheets("COSAGE output").Cells(j, 1)
        Shooter = Worksheets("COSAGE output").Cells(j, 4)
        munitName = Worksheets("COSAGE output").Cells(j, 6)
        Target = Worksheets("COSAGE output").Cells(j, 7)
        If (Munition = munitName) And (Record = "D" Or Record = "K") Then
            Sum = Sum + Worksheets("COSAGE output").Cells(j, 8)
            rrow6 = rrow6 + 1
            Worksheets("Munitions").Cells(rrow6, 1) = Munition
            Worksheets("Munitions").Cells(rrow6, 2) = Shooter
            Worksheets("Munitions").Cells(rrow6, 3) = Target
            Worksheets("Munitions").Cells(rrow6, 4) = Sum
        End If
    Next j
Next i

For k = 3 To rrow6
    Expended = 0
    Sum = 0
    munition1 = Worksheets("Munitions").Cells(k, 1)
    m = k
    Do Until Worksheets("Munitions").Cells(m, 1) <> munition1 Or Worksheets("Munitions").Cells(m, 1) = Empty
        Expended = Worksheets("Munitions").Cells(m, 4)
        m = m + 1
    Loop
    Worksheets("Munitions").Cells(k, 5) = Expended
    Worksheets("Munitions").Cells(k, 6) = Expended / 2
Next k

For m = 15 To Cosage
    Record = Worksheets("COSAGE output").Cells(m, 1)
    Init = Worksheets("COSAGE output").Cells(m, 13)
    Shots = Worksheets("COSAGE output").Cells(m, 8)
    If Record = "D" Or Record = "K" Then
        If Init <> 0 Then
            Worksheets("COSAGE output").Cells(m, 15) = (Shots / Init) / 2
        End If
    End If
Next m
'End Sub

End Sub
```

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APPENDIX C. MULTIPLE-PERIOD MODEL IMPLEMENTATION

The following is the Visual Basic multiple-period implementation of the analytical formulation introduced by Professors Gaver and Jacobs.

```
Private Sub MultiPeriodModel_Click()

'Sub CosageUpdate()
'This section of the macro populates worksheets "Red," "Red Update," "Blue,"
'and "Blue Update," and produces estimated shot rate (column O) and P(Kill)
'values (column P) in "COSAGE output"
Worksheets("COSAGE output").Range("o15:z20000") = ""

'Parameters
Cosage = 3521
initial = 215
Red = 112
Blue = 105
Reps = 16

For m = 15 To Cosage
    Record = Worksheets("COSAGE output").Cells(m, 1)
    Shooter = Worksheets("COSAGE output").Cells(m, 4)
    Munition = Worksheets("COSAGE output").Cells(m, 6)
    dirShots = Worksheets("COSAGE output").Cells(m, 8)
    indShots = Worksheets("COSAGE output").Cells(m, 14)
    initShtrs = Worksheets("COSAGE output").Cells(m, 13)
    If initShtrs <> "0" And initShtrs <> Empty Then
        If Record = "D" Or Shooter = "UMLRS" Or Munition = "PGM" Then
            ShotRate = 0.5 * (dirShots / initShtrs)
        ElseIf Record = "K" Then
            ShotRate = 0.5 * (indShots / initShtrs)
        End If
    End If
    Worksheets("COSAGE output").Cells(m, 15) = ShotRate
Next m

For i = 15 To Cosage
    Record = Worksheets("COSAGE output").Cells(i, 1)
    Shooter = Worksheets("COSAGE output").Cells(i, 4)
    Munit = Worksheets("COSAGE output").Cells(i, 6)
    Kills = Worksheets("COSAGE output").Cells(i, 9)
    dirShots = Worksheets("COSAGE output").Cells(i, 8)
```

```
    indShots = Worksheets("COSAGE output").Cells(i, 14)
    initTgts = Worksheets("COSAGE output").Cells(i, 12)
    If Record = "D" Or Shooter = "UMLRS" Or Munit = "PGM" Then
        If Kills > dirShots Then
            Kills = dirShots
        End If
        pKill = (Kills + (1 / Reps)) / (dirShots + (2 / Reps))
        Worksheets("COSAGE output").Cells(i, 16) = pKill
    ElseIf Record = "K" Then
        pKill = (Kills + (1 / Reps)) / ((indShots * initTgts) + (2 / Reps))
        Worksheets("COSAGE output").Cells(i, 16) = pKill
    End If
Next i

'End Sub

''''''''''''''''''''''''''''''''''''''''''''''''''''''''''''''''''''''''''
'Sub Update()
'This section of the macro implements the formulation described in Chapter 4, using Bayesian
'estimates to predict Red and Blue survivors for initial and successive 24-hour periods
Worksheets("Red Update").Range("c2:z20000") = ""
Worksheets("Red Update").Range("c2:z20000") = ""

'Parameters
Cosage = 3521
Red = 112
Blue = 105
Reps = 16
Updates = TextBox1.Value

For ttime = 1 To Updates 'User-defined number of 24-hour periods to model
    'Red Updates
    Worksheets("Red Update").Cells(2, ttime + 2) = "Time (t + " & ttime & ")"
    Worksheets("Blue Update").Cells(2, ttime + 2) = "Time (t + " & ttime & ")"

    For i = 3 To Red
        platform = Worksheets("Red Update").Cells(i, 1)
        Number = Worksheets("Red Update").Cells(i, ttime + 1)
```

```
    Survivors = Number
    For j = 15 To Cosage
        target = Worksheets("COSAGE output").Cells(j, 7)
        If target = platform Then
            Record = Worksheets("COSAGE output").Cells(j, 1)
            Shooter = Worksheets("COSAGE output").Cells(j, 4)
            Munit = Worksheets("COSAGE output").Cells(j, 6)
            Shot = Worksheets("COSAGE output").Cells(j, 8)
            ShotRate = Worksheets("COSAGE output").Cells(j, 15)
            P_kill = Worksheets("COSAGE output").Cells(j, 16)
            P_Survive = Worksheets("COSAGE output").Cells(j, 17)
            For b = 3 To Blue
                BlueShtr = Worksheets("Blue Update").Cells(b, 1)
                If BlueShtr = Shooter Then
                    NumShtrs = Worksheets("Blue Update").Cells(b, ttime + 1)
                    NumShots = NumShtrs * ShotRate
                    If Record = "D" Or Shooter = "UMLRS" Or Munit = "PGM" Then
                        If Number = 0 Then
                            Survivors = 0
                        Else:
                            Survivors = Survivors * ((1 - P_kill) ^ (NumShots / Number))
                        End If
                    ElseIf Record = "K" Then
                        If Number = 0 Then
                            Survivors = 0
                        Else:
                            Survivors = Survivors * ((1 - P_kill) ^ NumShots)
                        End If
                    End If
                End If
            Next b
        End If
    Next j
    Worksheets("Red Update").Cells(i, ttime + 2) = Survivors
Next i

'Blue Updates
For i = 3 To Blue
```

```
        platform = Worksheets("Blue Update").Cells(i, 1)
        Number = Worksheets("Blue Update").Cells(i, ttime + 1)
        Survivors = Number
        For j = 15 To Cosage
            target = Worksheets("COSAGE output").Cells(j, 7)
            If target = platform Then
                Record = Worksheets("COSAGE output").Cells(j, 1)
                Shooter = Worksheets("COSAGE output").Cells(j, 4)
                Shot = Worksheets("COSAGE output").Cells(j, 8)
                ShotRate = Worksheets("COSAGE output").Cells(j, 15)
                P_kill = Worksheets("COSAGE output").Cells(j, 16)
                P_Survive = Worksheets("COSAGE output").Cells(j, 17)
                For b = 3 To Red
                    RedShtr = Worksheets("Red Update").Cells(b, 1)
                    If RedShtr = Shooter Then
                        NumShtrs = Worksheets("Red Update").Cells(b, ttime + 1)
                        NumShots = NumShtrs * ShotRate
                        If Record = "D" Or Shooter = "UMLRS" Or Munit = "PGM" Then
                            If Number = 0 Then
                                Survivors = 0
                            Else:
                                Survivors = Survivors * ((1 - P_kill) ^ (NumShots / Number))
                            End If
                        ElseIf Record = "K" Then
                            If Number = 0 Then
                                Survivors = 0
                            Else:
                                Survivors = Survivors * ((1 - P_kill) ^ NumShots)
                            End If
                        End If
                    End If
                Next b
            End If
        Next j
        Worksheets("Blue Update").Cells(i, ttime + 2) = Survivors
    Next i
Next ttime
'End Sub
```

```
End Sub
'''''''''''''''''''''''''''''''''''''''''''''''''''''''''''''''''''''''''
Private Sub ShotsMacro_Click()

'Sub Shots()
'This section of the macro estimates the number of shots for each
'shooter/munition/target present, displaying the results on worksheet "Shots"
Worksheets("Shots").Range("a4:e20000") = ""

'Parameters
Cosage = 3521
RedStop = 1330
BlueStart = 1396
BlueStop = 3464
Red = 112
Blue = 105
Reps = 16
rrow = 3
ttime = TextBox1.Value

Worksheets("Shots").Cells(2, 3) = "Shots Expended for time (t + " & ttime & ")"
For i = 3 To Red
    platform = Worksheets("Red Update").Cells(i, 1)
    Survivors = Worksheets("Red Update").Cells(i, ttime + 2)
    For j = 15 To RedStop
        Record = Worksheets("COSAGE output").Cells(j, 1)
        If Record = "D" Or Record = "K" Then
            Shooter = Worksheets("COSAGE output").Cells(j, 4)
            Munition = Worksheets("COSAGE output").Cells(j, 6)
            target = Worksheets("COSAGE output").Cells(j, 7)
            ShotRate = Worksheets("COSAGE output").Cells(j, 15)
            If Shooter = platform Then
                rrow = rrow + 1
                TotShots = Survivors * ShotRate
                Worksheets("Shots").Cells(rrow, 1) = Shooter
                Worksheets("Shots").Cells(rrow, 2) = Survivors
                Worksheets("Shots").Cells(rrow, 3) = Munition
```

```
                Worksheets("Shots").Cells(rrow, 4) = target
                Worksheets("Shots").Cells(rrow, 5) = TotShots
            End If
        End If
    Next j

Next i

For i = 3 To Blue
    platform = Worksheets("Blue Update").Cells(i, 1)
    Survivors = Worksheets("Blue Update").Cells(i, ttime + 2)
    For j = BlueStart To BlueStop
        Record = Worksheets("COSAGE output").Cells(j, 1)
        If Record = "D" Or Record = "K" Then
            Shooter = Worksheets("COSAGE output").Cells(j, 4)
            Munition = Worksheets("COSAGE output").Cells(j, 6)
            target = Worksheets("COSAGE output").Cells(j, 7)
            ShotRate = Worksheets("COSAGE output").Cells(j, 15)
            If Shooter = platform Then
                rrow = rrow + 1
                TotShots = Survivors * ShotRate
                Worksheets("Shots").Cells(rrow, 1) = Shooter
                Worksheets("Shots").Cells(rrow, 2) = Survivors
                Worksheets("Shots").Cells(rrow, 3) = Munition
                Worksheets("Shots").Cells(rrow, 4) = target
                Worksheets("Shots").Cells(rrow, 5) = TotShots
            End If
        End If
    Next j

Next i
'End Sub
'''''''''''''''''''''''''''''''''''''''''''''''''''''''''''''''''''''''''
'Sub Totals()
'This section of the macro calculates total munitions expenditures by
'munition type, displaying the results on worksheet "Totals"
Worksheets("Totals").Range("a4:z20000") = ""
```

```
'Parameters
Cosage = 3521
RedStop = 1330
BlueStart = 1396
BlueStop = 3464
RedShots = 1179
BlueShots = 1180
ShotsTotal = 3109
Red = 112
Blue = 105
Reps = 16
rrow = 3

Worksheets("Totals").Cells(2, 3) = "Munitions Expended for time (t + " & ttime & ")"
For i = 901 To 1023
    Sum = 0
    Munition = Worksheets("Platforms").Cells(i, 4)
    For j = 4 To ShotsTotal
        Shooter = Worksheets("Shots").Cells(j, 1)
        Munit = Worksheets("Shots").Cells(j, 3)
        target = Worksheets("Shots").Cells(j, 4)
        TotShots = Worksheets("Shots").Cells(j, 5)
        If Munition = Munit Then
            Sum = Sum + TotShots
            rrow = rrow + 1
            Worksheets("Totals").Cells(rrow, 1) = Munition
            Worksheets("Totals").Cells(rrow, 2) = Shooter
            Worksheets("Totals").Cells(rrow, 3) = target
            Worksheets("Totals").Cells(rrow, 4) = Sum
        End If
    Next j
Next i

For k = 4 To rrow
    expended = 0
    ssum = 0
    munition1 = Worksheets("Totals").Cells(k, 1)
    m = k
```

```
    Do Until Worksheets("Totals").Cells(m, 1) <> munition1 Or Worksheets("Totals").Cells(m, 1) = Empty
        expended = Worksheets("Totals").Cells(m, 4)
        m = m + 1
    Loop
    Worksheets("Totals").Cells(k, 5) = expended
Next k
'End Sub

End Sub
```

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